

LETTER

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## Letter

# Effects of Cr<sup>4+</sup> ions on forming Ince–Gaussian modes in passively Q-switched microchip solid-state lasers

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**Abstract**

The effects of Cr<sup>4+</sup> ions on forming Ince–Gaussian (IG) laser modes in a tilted pumped Cr<sup>4+</sup>:YAG passively Q-switched (PQS) Nd:YAG microchip lasers has been studied experimentally and theoretically. The formation of Cr<sup>4+</sup>-ion domains in Cr<sup>4+</sup>:YAG crystal has been proposed based on the spatial distribution of Cr<sup>4+</sup> ions in Cr<sup>4+</sup>:YAG crystal. The spatial modulation effect of Cr<sup>4+</sup>-ion domains on selecting a transverse mode has been investigated based on their periodic distribution and nonlinear absorption of the Cr<sup>4+</sup> saturable absorber. The formation of IG modes in the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser under tilted pumping has been demonstrated theoretically with the spatial modulation of the Cr<sup>4+</sup>-ion domains in Cr<sup>4+</sup>:YAG crystal and the inversion population distribution, by taking account of the thermal lens effect. The IG modes selected theoretically in the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser are in good agreement with experimentally obtained IG modes. This work provides a theoretical model of the selection of IG modes with Cr<sup>4+</sup>-ion domains in a tilted beam pumped Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser to produce efficient, stable and controllable IG laser modes.

Keywords: Ince–Gaussian, microchip laser, passively Q-switched laser, Cr<sup>4+</sup>:YAG

(Some figures may appear in colour only in the online journal)

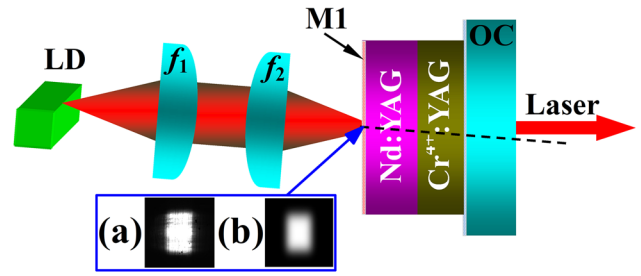
**1. Introduction**

Ince–Gaussian (IG) modes have been demonstrated for potential applications in the optical manipulation of microparticles, optical trapping and investigations into the spatial behavior of phase singularities [1–4]. Laser-diode pumped IG mode solid-state lasers have been obtained using the method of breaking the symmetry of the laser cavity [5–7]. However, these solid-state lasers are a continuous wave (CW) operation, their output power is low and the lasers are less efficient owing to the misalignment of the laser cavity. High-order IG modes Cr,Nd:YAG self-Q-switched (SQS) nanosecond

microchip lasers with a peak power of over 1 kW have been demonstrated by applying a tilted pump beam [8, 9], and various high-order IG modes have also been obtained in the Cr<sup>4+</sup>:YAG passively Q-switched (PQS) Nd:YVO<sub>4</sub> microchip laser under the tilted pumping [10]. It has been demonstrated that the tilted pump scheme is an effective method to break the symmetry of the plane-parallel microchip cavity for generating IG mode lasers. However, the generation mechanism of the IG laser modes in SQS and PQS microchip lasers with a plane-parallel cavity is not clear. Usually, the generation mechanism of the high-order transverse mode can be categorized as follows: phase modulation [11–14], gain control

[15–17] and loss control [18, 19]. The phase modulation requires the use of spatial light modulation devices such as the liquid crystal spatial light modulator [11, 12], the kinoform phase elements [13] and the digital micromirror device [14]. The IG mode can also be obtained in the solid-state laser with a plano-spherical resonator by off-axis pumping, which is a method of gain control [15, 16]; however, this method does not work for microchip lasers with a plane-parallel cavity, because the symmetry of the plane-parallel cavity is not broken under the off-axis pumping. In addition, the laser mode can be changed by controlling the gain distribution inside the laser crystal [20–23]. Loss control is a method of introducing diffraction loss by using opaque lines [18], a cross hair [5] or a spot-defect mirror [19] in the solid-state laser. However, none of these mechanisms can completely explain the generation mechanism of the IG mode in the PQS microchip laser with a plane-parallel cavity. Besides the laser cavity, the pump beam profile and the saturable absorber also affect the oscillation of the transverse laser modes. Various transverse laser modes have been obtained by changing the pumping beam profiles, for example, the Laguerre–Gaussian (LG), Hermite–Gaussian (HG) and IG modes have been obtained in the Nd:YAG PQS microchip laser pumped with different decentered Gaussian beams [21]. The effects of the Cr<sup>4+</sup>:YAG saturable absorbers on generating transverse laser modes in PQS solid-state lasers have been studied in recent decades [24–27]. It has been found that the saturable absorber Cr<sup>4+</sup>:YAG crystal has the effect of the beam narrowing; even though the intracavity aperture is larger than the size of the fundamental mode spot, the fundamental mode was still obtained in a PQS Nd:YAG laser [25]. The high-order LG modes were obtained in a PQS Nd:YAG laser with the help of a saturable absorber combined with a suitable intracavity aperture and a phase element in the cavity. There is a significant difference in the laser intensity distribution between the non-Q-switched operation and the PQS operation, which shows that saturable absorber plays an important role in the transverse-mode selection [26]. Four phase-locked Gaussian beams are generated in the Cr<sup>4+</sup>:YAG PQS laser due to the dynamic mode filter effect of the saturable absorber [27]. However, the exact process of laser mode selection by the Cr<sup>4+</sup>:YAG saturable absorber is still unclear in the PQS microchip laser with a plane-parallel cavity. Therefore, the generation mechanism of the IG laser modes in the PQS microchip laser is worth investigating.

In this paper, three comparative experiments (I: CW Nd:YAG microchip laser, II: Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser, III: Cr,Nd:YAG SQS microchip laser) were designed to study the generation mechanism of the IG laser modes in microchip lasers. A single emitter laser diode was used as the pump source, since the rectangular profile from the single emitter laser diode has better mode matching with the IG mode than the circular Gaussian pump beam. The formation of Cr<sup>4+</sup>-ion domains as a spatial light filter to force IG modes oscillation in the PQS microchip laser were investigated theoretically and a mechanism of formation of IG modes in the PQS microchip laser was proposed based on the interaction between asymmetrical saturated inversion population



**Figure 1.** The experimental setup of the tilted beam pumped Cr<sup>4+</sup>:YAG passively Q-switched Nd:YAG microchip laser for IG mode generation.  $f_1$  and  $f_2$  are the focus lenses with an 8 mm focal length, M1 is the rear cavity mirror, OC is the output coupler. The measured and theoretically calculated pump beam profiles at the focus spot are shown in insets (a) and (b), respectively.

distribution inside the laser crystal and the mode selection of the nonlinear absorption of Cr<sup>4+</sup>-ion domains in Cr<sup>4+</sup>:YAG crystal. The theoretically formed IG modes in the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser are in good agreement with experimentally obtained IG modes.

## 2. Experiments

The experimental setup for studying the mechanism of generating IG modes in the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser pumped with a tilted focused beam from a single-emitter laser diode is shown in figure 1. Based on our previous works on direct generation of IG modes in a Cr,Nd:YAG SQS microchip laser under tilted beam pumping [8, 9], by taking into account the exponential decay of the absorbed pump power along the crystal length, the thicknesses of the Nd:YAG crystal and Cr,Nd:YAG crystal were chosen to be 1.8 mm to achieve the same gain in the comparison experiments. The majority of the incident pump power (about 72%) is absorbed by a 1.8 mm thick Nd:YAG crystal or Cr,Nd:YAG crystal doped with 1 at.% Nd<sup>3+</sup> ions (the absorption coefficient was measured to be 7 cm<sup>-1</sup> at 808 nm). The extra absorbed pump power by increased crystal length exceeding 1.8 mm is not sufficient to provide gain for laser oscillation. **And the losses for the laser are increased by increasing the length of the Cr,Nd:YAG crystal or the Nd:YAG crystal. Therefore, there is a tradeoff for selecting the thickness of the Cr,Nd:YAG or Nd:YAG crystals to achieve high performance of the Cr,Nd:YAG SQS microchip laser or the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser.** One surface of the Nd:YAG crystal facing pump beam was coated with high reflection (HR) at 1064 nm and antireflection (AR) at 808 nm, and the other surface was coated with AR at 1064 nm to reduce the intracavity loss. A 1.5 mm-thick Cr<sup>4+</sup>:YAG crystal (0.01 at.% Cr doping concentration) with 95% initial transmission ( $T_0$ ) was used as a saturable absorber. A 1.8 mm-thick Cr,Nd:YAG SQS crystal codoped with 1 at.% Nd<sup>3+</sup> ions and 0.01 at.% Cr ions ( $T_0 = 94%$ ) was used to compare the laser performance with the CW and PQS Nd:YAG microchip lasers. The same Cr doping concentration in Cr<sup>4+</sup>:YAG and Cr,Nd:YAG crystals was chosen to avoid the effect of Cr<sup>4+</sup> ion concentration on the formation of the IG mode in PQS microchip lasers. And the small

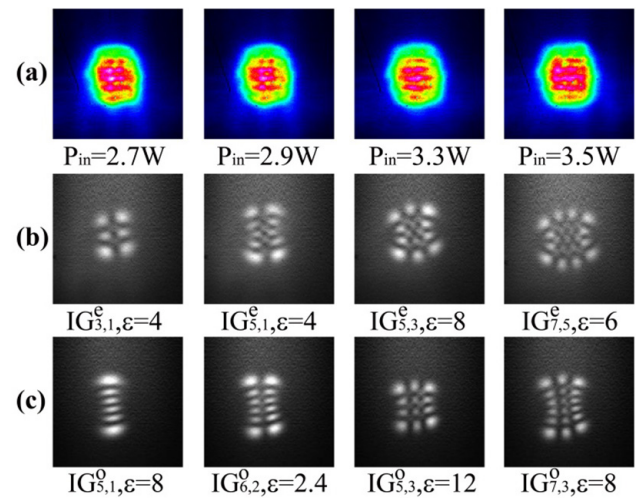
difference between the  $T_0$  of the  $\text{Cr}^{4+}$ :YAG crystal and the Cr,Nd:YAG crystal can be neglected for the investigation of IG modes in the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser and the Cr,Nd:YAG SQS microchip laser. A plane-parallel output coupling mirror with a reflection of 95% at 1064 nm was used in the laser experiments. The cavity length of the CW and Cr,Nd:YAG SQS microchip laser was 1.8 mm, while the cavity length of the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser is 3.3 mm. A single-emitter laser-diode working at 808 nm (emitting cross section:  $1 \times 50 \mu\text{m}^2$ ) was used as the pump source. The fast-axis divergence angle of the laser diode was reduced to  $10^\circ$  using a micro lens at the output facet of the laser diode. Two lenses with a focal length of 8 mm were used to collimate and focus the pump beam from the laser diode. The measured footprint at the focused spot was  $100 \mu\text{m} \times 150 \mu\text{m}$  ( $x$  direction  $\times y$  direction) after collimating and focusing optics, as shown in figure 1(a). The pump beam can be described by a super-Gaussian function,

$$E(x, y, z) = E_0 \cdot \exp \left[ - \left( \frac{x^4}{w_x(z)^4} + \frac{y^4}{w_y(z)^4} \right) \right], \quad (1)$$

where  $E_0$  is a constant,  $w_x(z)$  and  $w_y(z)$  are the width of the pump beam along the  $x$ -axis and the  $y$ -axis at the distance  $z$ , respectively. Figure 1(b) gives the theoretically calculated profile of the focused pump beam, which is in good agreement with the experimentally measured pump beam profile, as shown in figure 1(a). The incident pump beam was tilted  $3^\circ$  away from the laser direction for generating IG modes. The characteristics of the laser pulse were recorded using a fast InGaAs photodiode and a digital oscilloscope (Tektronix TDS6604). The output laser beam profiles were monitored and recorded with a laser beam quality analyzer (Thorlabs BC106-VIS).

### 3. Experimental results

Firstly, the transverse intensity patterns in the CW Nd:YAG microchip laser, the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser and the Cr,Nd:YAG SQS microchip laser have been investigated by normal incident to the rectangular pump beam on the crystals. Unfortunately, no IG modes were observed in normally pumped PQS and SQS microchip lasers. The output profiles are fundamental mode or top-hat profiles depending on the applied pump power, which are similar to those obtained in the single-emitter laser diode end-pumped Yb:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser [28]. The transverse intensity patterns generated in the tilted beam pumped CW Nd:YAG microchip laser, Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser and Cr,Nd:YAG SQS microchip laser have been measured at different incident pump powers ( $P_{\text{in}}$ ) with a beam profiler. The  $\text{TEM}_{00}$  mode laser was observed in the CW Nd:YAG microchip laser under tilted beam pumping when the  $P_{\text{in}}$  was above the pump power threshold ( $P_{\text{th}}$ ) and lower than 1.5 W. The output laser transverse beam profile tends to be a rectangular profile with further increase in  $P_{\text{in}}$  because the pump power intensity within the rectangular pump beam is sufficient to support multimode laser oscillation. The rectangular



**Figure 2.** Experimentally obtained laser transverse patterns of (a) CW Nd:YAG microchip laser, (b) Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser, and (c) Cr,Nd:YAG SQS microchip laser at different incident pump powers.

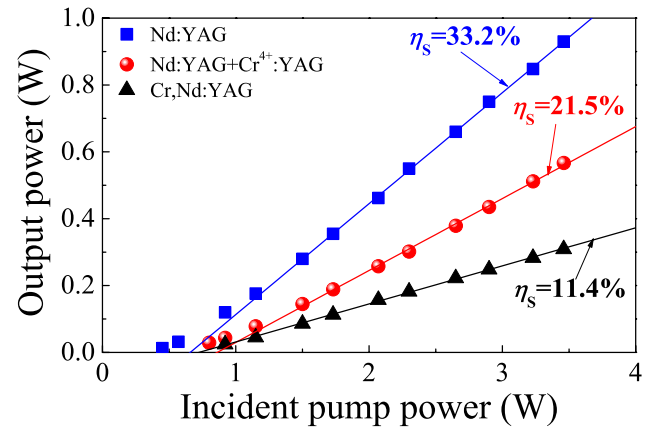
transverse intensity profile was maintained with a further increase in the  $P_{\text{in}}$ , as shown in figure 2(a). The transverse intensity distribution of the laser beam is close to a top-hat profile and no IG modes were observed in the CW Nd:YAG microchip laser within the whole pump power range.

When the  $\text{Cr}^{4+}$ :YAG crystal was sandwiched between the Nd:YAG crystal and the output coupler, a Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser was constructed. The incident pump power threshold increased to 0.8 W owing to the insert loss of the  $\text{Cr}^{4+}$ :YAG crystal. The  $\text{TEM}_{00}$  mode laser was observed when the  $P_{\text{in}}$  was within the range 0.8 W to 1.5 W. The IG modes were observed when the  $P_{\text{in}} > 1.5$  W. The index of the IG modes increases with the  $P_{\text{in}}$ . Figure 2(b) gives some typical IG mode distribution obtained in the PQS Nd:YAG microchip laser at different incident pump powers. The top-hat rectangular beam profile of the CW Nd:YAG microchip laser is changed to  $\text{IG}_{3,1}^\epsilon$  ( $\epsilon = 4$ ) mode for the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser at  $P_{\text{in}} = 2.7$  W. Upon further increase of the  $P_{\text{in}}$ , the IG mode oscillation was maintained and the index of the IG mode increased. The  $\text{IG}_{5,1}^\epsilon$  ( $\epsilon = 4$ ) mode was obtained at  $P_{\text{in}} = 2.9$  W. The Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser oscillated in  $\text{IG}_{5,3}^\epsilon$  ( $\epsilon = 8$ ) mode at  $P_{\text{in}} = 3.3$  W. The  $\text{IG}_{7,5}^\epsilon$  ( $\epsilon = 6$ ) mode was observed at  $P_{\text{in}} = 3.5$  W. The similar phenomena of generating IG modes were also observed in the Cr,Nd:YAG SQS microchip laser. The  $P_{\text{th}}$  increased to 0.92 W for the Cr,Nd:YAG SQS microchip laser, which was attributed to the high intracavity loss induced by codoping  $\text{Nd}^{3+}$  ions and  $\text{Cr}^{4+}$  ions in the YAG host crystal. More defects were introduced in the Cr,Nd:YAG crystal because compensation charges such as  $\text{Ca}^{2+}$  ions or  $\text{Mg}^{2+}$  ions were needed to form  $\text{Cr}^{4+}$  ions in the Cr,Nd:YAG crystal. The odd IG modes were obtained in the Cr,Nd:YAG SQS microchip laser when  $P_{\text{in}}$  was higher than 2 W. Figure 2(c) gives some typical transverse mode distribution obtained in the Cr,Nd:YAG SQS microchip laser at different  $P_{\text{in}}$ 's. Owing to more loss being introduced in the Cr,Nd:YAG crystal, the transverse profiles of the observed IG modes were totally different from those observed in the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser at the same  $P_{\text{in}}$ .

The  $IG_{5,1}^0$  ( $\varepsilon = 8$ ),  $IG_{6,2}^0$  ( $\varepsilon = 2.4$ ),  $IG_{5,3}^0$  ( $\varepsilon = 12$ ), and  $IG_{7,3}^0$  ( $\varepsilon = 8$ ) were obtained in the Cr,Nd:YAG SQS microchip laser at  $P_{in} = 2.7, 2.9, 3.3$  and  $3.5$  W, respectively, as shown in figure 2(c).

Compared to the rectangular top-hat transverse profiles obtained in the CW Nd:YAG microchip laser pumped with a tilted pump beam as shown in figure 2(a), the tilted beam pumped Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser and Cr,Nd:YAG SQS microchip laser oscillated in IG modes, as shown in figures 2(b) and (c). The breaking of the cavity symmetry in the CW Nd:YAG microchip laser under tilted beam pumping is not sufficient to force the CW Nd:YAG microchip laser to oscillate in IG modes. However, IG modes obtained in the tilted beam pumped Nd:YAG/Cr<sup>4+</sup>:YAG PQS and Cr,Nd:YAG SQS microchip lasers clearly show that the Cr<sup>4+</sup>:YAG saturable absorber plays a key role in the formation of IG modes in Nd:YAG/Cr<sup>4+</sup>:YAG PQS and Cr,Nd:YAG SQS microchip lasers. Therefore, the possible IG mode formation mechanism in the tilted beam pumped Nd:YAG/Cr<sup>4+</sup>:YAG PQS and Cr,Nd:YAG SQS microchip lasers is that the Cr<sup>4+</sup> saturable absorber acts as the spatial light filter to select the suitable IG modes in the deformed pumped area inside the gain medium. The deformed pump beam area inside the gain medium induced by the tilted beam pumping provides an arena for possible IG mode oscillation selected by the nonlinear absorption of Cr<sup>4+</sup> ions.

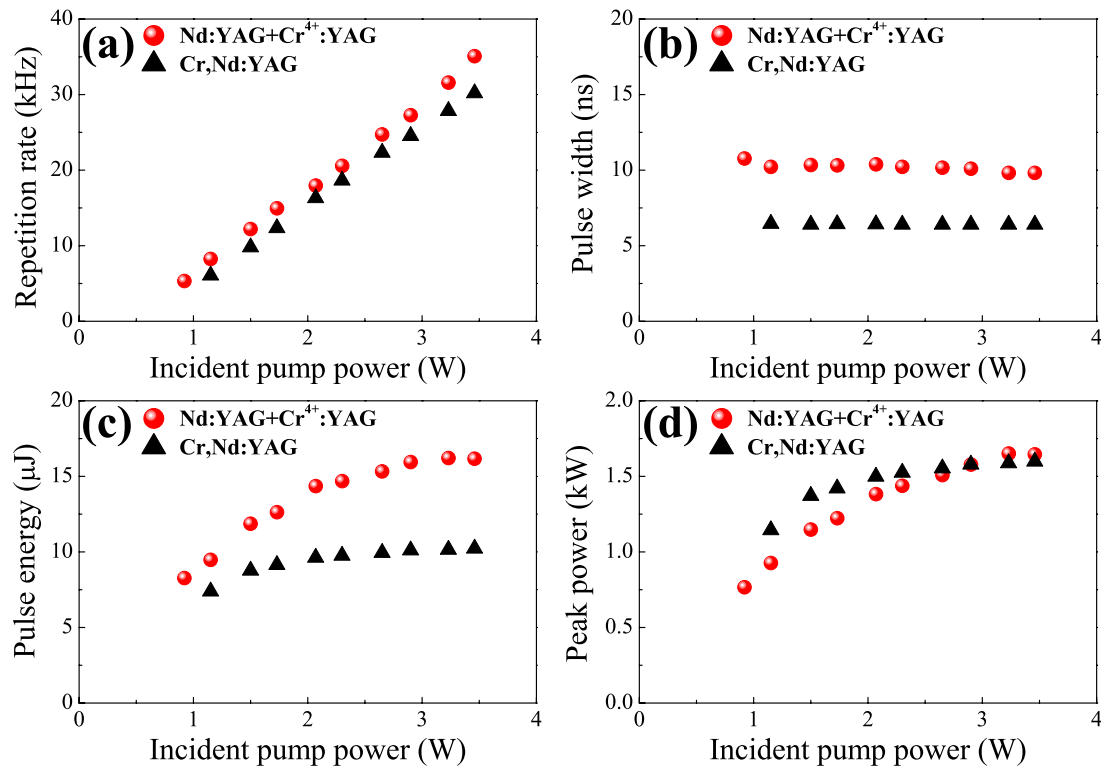
The Cr<sup>4+</sup> ions saturable absorbers do not only determine the formation of IG modes in the Nd:YAG/Cr<sup>4+</sup>:YAG PQS and Cr,Nd:YAG SQS microchip lasers, but also have strong effects on the performance of Nd:YAG/Cr<sup>4+</sup>:YAG PQS and Cr,Nd:YAG SQS microchip lasers. The laser performance of the CW Nd:YAG microchip laser, Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser and Cr,Nd:YAG SQS microchip laser was measured. Figure 3 shows the output power of the CW Nd:YAG microchip laser and Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser as a function of  $P_{in}$ , together with the output power of the Cr,Nd:YAG SQS microchip laser for comparison. The pump power thresholds of the CW Nd:YAG microchip laser, Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser, and Cr,Nd:YAG SQS microchip laser are 0.45 W, 0.8 W, and 0.92 W, respectively. The increase in the incident pump power threshold for the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser was attributed to the insert losses of the Cr<sup>4+</sup>:YAG crystal in the laser cavity. The high incident pump power threshold of the Cr,Nd:YAG SQS microchip laser was caused by the severe distortion of crystalline lattices and defects with codoped Cr ions and Nd<sup>3+</sup> ions in the same host crystal. It is well known that the Nd<sup>3+</sup> ions in Nd:YAG crystal are formed by substituting the Y<sup>3+</sup> ions. And it has been demonstrated that the Cr<sup>4+</sup> ions in Cr<sup>4+</sup>:YAG crystal are formed by substituting the tetrahedral Al<sup>3+</sup> sites, compensating charges such as Ca<sup>2+</sup> or Mg<sup>2+</sup> ions which are required to substitute the dodecahedral Y<sup>3+</sup> sites. Because the ionic radii of Nd<sup>3+</sup> ions and Y<sup>3+</sup> ions are different, and the ionic radii of Cr ions and Al<sup>3+</sup> ions are also different, the formation of Nd<sup>3+</sup> ions and Cr<sup>4+</sup> ions simultaneously in Cr,Nd:YAG crystal causes aggressive distortion of crystalline lattices and more defects compared to the formation of Nd<sup>3+</sup>



**Figure 3.** The output power as a function of the incident pump power for the CW Nd:YAG microchip laser, Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser and Cr,Nd:YAG SQS microchip laser.

ions in Nd:YAG crystal or Cr<sup>4+</sup> ions in Cr<sup>4+</sup>:YAG crystal. And the compensating charges also induce distortion and extra losses in growth of Cr,Nd:YAG crystal. Therefore, for the same doping concentration of Nd<sup>3+</sup> ions and Cr ions, the defects and distortion in Cr,Nd:YAG crystal are more severe than that in Nd:YAG crystal and Cr<sup>4+</sup>:YAG crystal. For Nd:YAG, Cr<sup>4+</sup>:YAG and Cr,Nd:YAG crystals doped with 1 at.% Nd<sup>3+</sup> ions and 0.01 at.% Cr ions used in the laser experiments, the threshold pump power for the Cr,Nd:YAG SQS microchip laser is higher than for the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser. The output power increases linearly with  $P_{in}$  for the CW Nd:YAG microchip laser. The slope efficiency ( $\eta_s$ ) is measured to be 33.2%. The maximum output power of 0.93 W was obtained at  $P_{in} = 3.5$  W, the optical-to-optical efficiency ( $\eta_{o-o}$ ) was 27%. The average output power of the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser and the Cr,Nd:YAG SQS microchip laser increases linearly with  $P_{in}$ . The slope efficiencies ( $\eta_s$ ) drop to 21.5% and 11.4% for the PQS Nd:YAG microchip laser and Cr,Nd:YAG SQS microchip laser, respectively. The maximum average output power of 0.57 W and 0.31 W was obtained at  $P_{in} = 3.5$  W for the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser and the Cr,Nd:YAG SQS microchip laser, respectively. The optical-to-optical efficiencies were 16.3% and 8.8%, respectively. The output power from these three lasers is not saturated and can be further scaled by applying high pump power. The drop of the average output power in the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser is attributed to the insert losses of Cr<sup>4+</sup>:YAG saturable absorber in the laser cavity. The further degraded performance of the Cr,Nd:YAG SQS microchip laser is attributed to severe distortion of the crystalline lattices and defects introduced in codoped Cr,Nd:YAG crystal.

The IG modes obtained under the same pumping conditions in the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser and the Cr,Nd:YAG SQS microchip laser are different. This is due to the different doping concentration of Cr<sup>4+</sup> ions and cavity length. The different Cr<sup>4+</sup> ion doping concentrations determine the different sizes of the Cr<sup>4+</sup>-ion domains and form different IG modes. At the same time, the different Cr<sup>4+</sup>-ion domains have a great effect on the laser performance of the



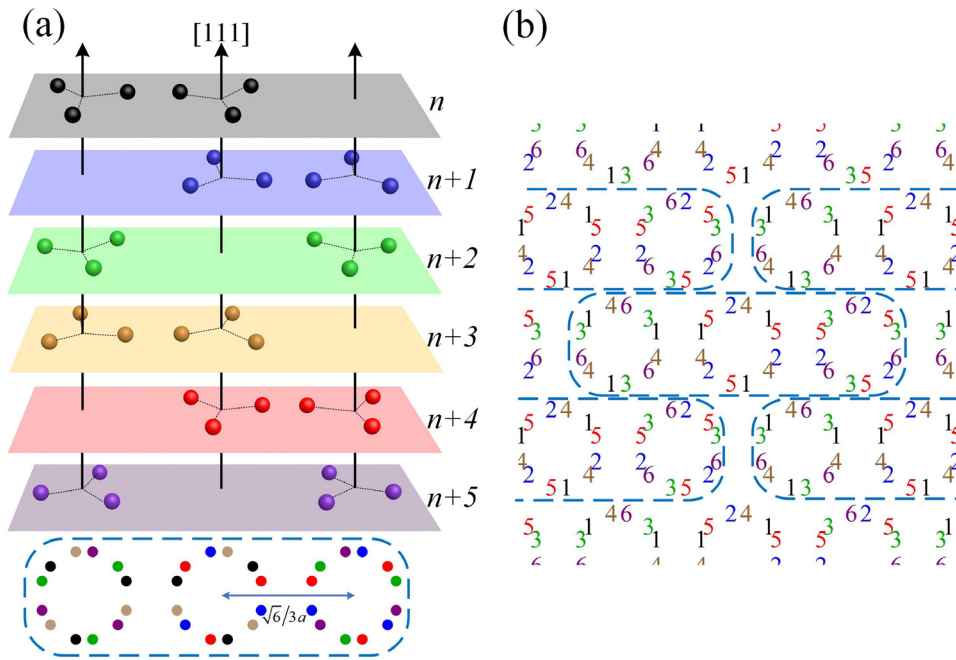
**Figure 4.** The repetition rate (a), pulse width (b), pulse energy (c) and peak power (d) as a function of the incident pump power.

Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser and the Cr,Nd:YAG SQS microchip laser. The output pulse characteristics are also affected by factors such as defects of the crystalline lattice and cavity length. The defects and distortion of the crystalline lattice in Cr,Nd:YAG crystal is more severe than in Cr<sup>4+</sup>:YAG crystal, and degrades the performance of the Cr,Nd:YAG SQS microchip laser. The repetition rate and the pulse energy of the Cr,Nd:YAG SQS microchip laser is less than those obtained in Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser. The repetition rate increases with  $P_{in}$  for both PQS and SQS microchip lasers, as shown in figure 4(a). When  $P_{in}$  is 3.5 W, the highest repetition rates of 38.3 kHz and 27.9 kHz were obtained for PQS and SQS microchip lasers, respectively. The maximum pulse energy is about 15  $\mu$ J and 10  $\mu$ J for PQS and SQS microchip lasers, respectively. In the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser, the pulse energy increases slowly with the incident pump power when  $P_{in}$  is less than 2.7 W and tends to be saturated with further increase in  $P_{in}$ , as shown in figure 4(c). This is caused by the full saturation of the Cr<sup>4+</sup>:YAG saturable absorber under high intracavity laser intensity at high pump power level, and the energy stored in the Nd:YAG crystal is fully extracted, so the pulse energy remains constant. In the Cr,Nd:YAG SQS microchip laser, the pulse energy remained essentially constant with the increase in  $P_{in}$ . It is well known that the pulse width of passively Q-switched lasers is mainly governed by the initial transmission of the saturable absorber and cavity length. For the same initial transmission of the saturable absorber, the shorter the cavity length, the shorter the pulse width that is generated. For the same cavity length, the higher the initial transmission of the saturable absorber, the wider the output pulse. Because the cavity

length ( $L_C = 1.8$  mm) of the Cr,Nd:YAG SQS microchip laser is shorter than that ( $L_C = 3.3$  mm) of the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser, and the  $T_0$  for both cases are comparable, the pulse width of the PQS microchip laser is mainly determined by the cavity length. The pulse width of the Cr,Nd:YAG SQS microchip laser is shorter than that of the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser. The pulse widths remained substantially constant (10 ns for the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser and 6.5 ns for the Cr,Nd:YAG SQS microchip laser) with the increase in  $P_{in}$ , as shown in figure 4(b). The short pulse width obtained in the Cr,Nd:YAG SQS microchip laser is beneficial for increasing the peak power of the Cr,Nd:YAG SQS microchip laser. And the peak power of the Cr,Nd:YAG SQS microchip laser is improved with short pulse width. The variation in the peak powers for PQS and SQS microchip lasers with  $P_{in}$  is similar to the variation of the pulse energy with  $P_{in}$ . The highest peak power of 1.5 kW is achieved for both PQS and SQS microchip lasers, as shown in figure 4(d). Therefore, a short laser cavity in the PQS microchip laser is beneficial for generating short pulses with high peak power. The performance of the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser such as short pulse width and high peak power can be further enhanced by applying thin Cr<sup>4+</sup>:YAG crystal doped with high Cr concentration to decrease initial transmission.

#### 4. Theoretical analysis of forming IG modes in Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser

Based on the experimental results for the CW Nd:YAG microchip laser, Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser and



**Figure 5.** The spatial distribution of the tetrahedral site  $\text{Al}^{3+}$  ions in a  $[111]$ -cut YAG crystal. (a) The distribution of tetrahedral  $\text{Al}^{3+}$  sites in six consecutive  $(111)$  planes and the projection of the tetrahedral  $\text{Al}^{3+}$  sites from six consecutive  $(111)$  planes, each dot representing a tetrahedral  $\text{Al}^{3+}$  site; (b) the periodical distribution of the projection of the tetrahedral  $\text{Al}^{3+}$  sites in six different  $(111)$  planes, number  $n$  representing the  $n$ th plane where tetrahedral  $\text{Al}^{3+}$  site is located.

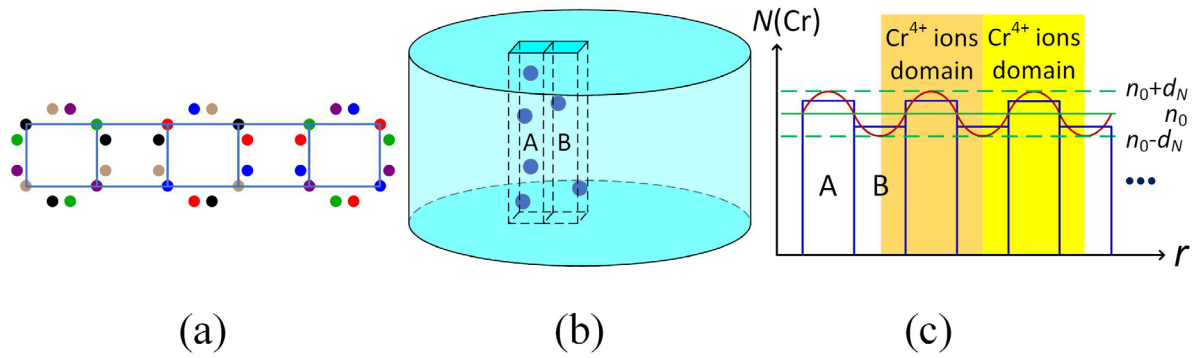
Cr,Nd:YAG SQS microchip laser, the formation of IG modes in microchip lasers under tilted beam pumping is determined by the  $\text{Cr}^{4+}$  ions saturable absorber. The formation of IG modes in a tilted beam pumped passively Q-switched microchip laser is governed by the interaction between the deformed pump area and the distribution of  $\text{Cr}^{4+}$  ions. The nonlinear absorption of the  $\text{Cr}^{4+}$  ions saturable absorber strongly depends on the laser intensity, and the possible oscillation of IG modes in the tilted beam pumped Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser strongly relies on the intracavity laser intensity distribution. The intracavity laser intensity distribution is determined by the inversion population distribution provided with the pump beam applied on the gain medium. The intracavity laser intensity is proportional to the inversion population provided by the pump power. And the intracavity laser intensity distribution is also strongly affected by the gain saturation effect, thermal lens effect and gain guiding effect. Therefore, the distribution of  $\text{Cr}^{4+}$  ions in YAG and the deformed pump area inside the gain medium, taking into account the asymmetrical thermal lens effect, should be considered in illustrating the formation of IG modes in PQS microchip lasers.

It is well known that the  $\text{Cr}^{4+}$ :YAG crystal belongs to the cubic system [29]. In YAG crystal there are two kinds of lattice sites for  $\text{Al}^{3+}$  ions, the tetrahedral  $\text{Al}^{3+}$  site and the octahedral  $\text{Al}^{3+}$  site. The  $\text{Cr}^{4+}$  ions in YAG crystal are formed by substituting the tetrahedral  $\text{Al}^{3+}$  sites; compensating charges such as  $\text{Ca}^{2+}$  or  $\text{Mg}^{2+}$  ions are required to substitute the dodecahedral Y sites [30]. Owing to the cubic symmetry, the distribution of  $\text{Cr}^{4+}$  ions in YAG crystal can be considered as the distribution of tetrahedral  $\text{Al}^{3+}$  sites in YAG crystal. The tetrahedral  $\text{Al}^{3+}$  ions distribute regularly and uniformly in the  $(111)$  plane of YAG crystal. Therefore, the distribution of  $\text{Cr}^{4+}$  ions

in  $\text{Cr}^{4+}$ :YAG crystal can be illustrated by stating the distribution of the tetrahedral  $\text{Al}^{3+}$  ions in YAG crystal. In a  $(111)$  plane of YAG crystal, an equilateral triangle is formed by connecting the adjacent tetrahedral  $\text{Al}^{3+}$  sites. The distribution of the tetrahedral  $\text{Al}^{3+}$  sites on the  $(111)$  planes has been studied and it has been found that the relative distribution of the tetrahedral  $\text{Al}^{3+}$  sites repeats itself every six consecutive  $(111)$  planes. In other words, the location of tetrahedral  $\text{Al}^{3+}$  sites on the  $n$ th  $(111)$  plane is consistent with the locations of tetrahedral  $\text{Al}^{3+}$  sites on the  $n + 6$ th  $(111)$  plane. Figure 5 shows the distribution of the tetrahedral  $\text{Al}^{3+}$  sites for six consecutive  $(111)$  planes, each dot in figure 5(a) representing a tetrahedral  $\text{Al}^{3+}$  site and the number  $n$  in figure 5(b) representing the  $n$ th plane where the tetrahedral  $\text{Al}^{3+}$  site is located. The distance  $d$  between each two consecutive  $(111)$  planes is  $\sqrt{3}a/12$ , where  $a$  is the lattice constant of YAG crystal, so the distance  $L$  between the  $n$ th  $(111)$  plane and the  $n + 6$ th  $(111)$  plane is  $\sqrt{3}a/2$ . The tetrahedral  $\text{Al}^{3+}$  sites are periodically distributed in space, and the sites distribution of one cycle is shown in figure 5(a). The adjacent tetrahedral  $\text{Al}^{3+}$  sites within six  $(111)$  planes form a circular distribution on the projection of the  $(111)$  plane, and the projection of the tetrahedral  $\text{Al}^{3+}$  sites in the dashed area is taken as one cycle, as shown in figure 5(a). The distance between the centers of adjacent rings is  $\sqrt{6}a/3$ .

The concentration of  $\text{Cr}^{4+}$  ions in the  $\text{Cr}^{4+}$ :YAG crystal is usually determined by measuring the initial transmission of  $\text{Cr}^{4+}$ :YAG crystal of a certain length. The initial transmission of  $\text{Cr}^{4+}$ :YAG crystal is related to the  $\text{Cr}^{4+}$  ion concentration and the length of the  $\text{Cr}^{4+}$ :YAG crystal, and can be expressed as [31]:

$$T_0 = \exp(-n_0 \sigma_{gs} l_s), \quad (2)$$



**Figure 6.** The formation and distribution of the  $\text{Cr}^{4+}$ -ion domains. (a) The distribution of the projection of the tetrahedral  $\text{Al}^{3+}$  sites in six different (111) planes. (b) The number of  $\text{Cr}^{4+}$  ions in the A and B regions in the  $\text{Cr}^{4+}$ :YAG crystal, the blue solid dots indicate the  $\text{Cr}^{4+}$  ions. (c) The trend of  $\text{Cr}^{4+}$  ion doping concentration along the  $r$  axis, where  $r$  represents the horizontal axis,  $N(\text{Cr})$  represents the doping concentration of  $\text{Cr}^{4+}$  ions,  $n_0$  indicates the average doping concentration of  $\text{Cr}^{4+}$  ions in the entire  $\text{Cr}^{4+}$ :YAG crystal, and  $d_N$  represents the fluctuation range of the doping concentration of  $\text{Cr}^{4+}$  ions.

where  $n_0$  is the total concentration of  $\text{Cr}^{4+}$  ions in the  $\text{Cr}^{4+}$ :YAG crystal,  $\sigma_{\text{gs}}$  is the ground-state absorption cross-section of  $\text{Cr}^{4+}$ :YAG crystal, and  $l_s$  is the length of the  $\text{Cr}^{4+}$ :YAG crystal. The total concentration of  $\text{Cr}^{4+}$  ions in the  $\text{Cr}^{4+}$ :YAG crystal can be expressed as:

$$n_0 = \frac{-\ln T_0}{\sigma_{\text{gs}} l_s}. \quad (3)$$

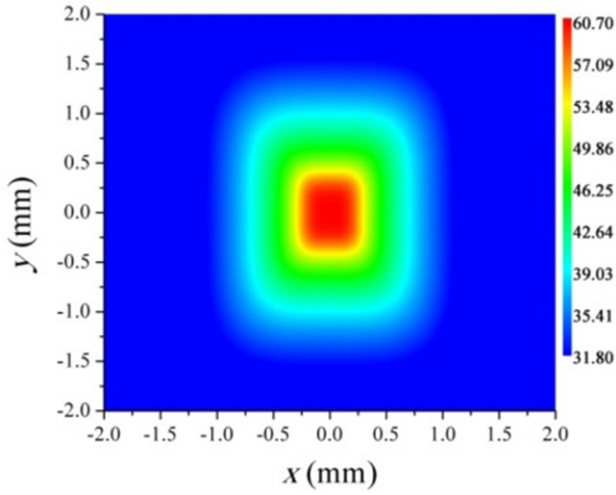
For the  $\text{Cr}^{4+}$ :YAG crystal with  $T_0 = 95\%$  used in the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser, the  $\text{Cr}^{4+}$  ion concentration,  $n_0$ , was estimated to be  $7.95 \times 10^{16} \text{ cm}^{-3}$  with the parameters of  $\sigma_{\text{gs}} = 4.3 \times 10^{-18} \text{ cm}^2$ , and  $l_s = 0.15 \text{ cm}$ . It is also known that the total particle density in the YAG crystal is  $9.2 \times 10^{22} \text{ cm}^{-3}$ , so that there are about  $1.16 \times 10^6$  ions around one  $\text{Cr}^{4+}$  ion in  $\text{Cr}^{4+}$ :YAG crystal under the assumption of uniform distribution of  $\text{Cr}^{4+}$  ions. We assume that the space occupied by each  $\text{Cr}^{4+}$  ion is a cube in the case of uniform doping. The distance between any two nearest adjacent  $\text{Cr}^{4+}$  ions is  $2.33 \times 10^{-8} \text{ m}$  ( $\approx 19a$ ) under the uniform distribution of  $\text{Cr}^{4+}$  ions in a  $\text{Cr}^{4+}$ :YAG crystal with  $T_0 = 95\%$ .

It is well known that the dimension of a single ion is on the nanometer scale and does not affect the formation of modes. The role of  $\text{Cr}^{4+}$ :YAG crystal in laser mode selection is caused by the combination of numerous  $\text{Cr}^{4+}$  ions to form  $\text{Cr}^{4+}$ -ion domains of a size (micrometers in diameter) comparable to the laser wavelength. Therefore, the formation of  $\text{Cr}^{4+}$ -ion domains along the laser direction (e.g.  $\langle 111 \rangle$  direction) in  $\text{Cr}^{4+}$ :YAG crystal is proposed for explaining IG mode oscillation in tilted pumped Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip lasers. A network along the laser direction is formed with adjacent  $\text{Cr}^{4+}$ -ion domains in  $\text{Cr}^{4+}$ :YAG crystal, which acts as a spatial light filter in the formation of IG mode oscillation. Here, the formation of IG modes with  $\text{Cr}^{4+}$ -ion domains is elucidated. Although the tetrahedral  $\text{Al}^{3+}$  sites are periodically distributed, they are not evenly spaced with the same interatomic distance. We divide some adjacent regions of the same size on the projection of the (111) plane, and the number of the tetrahedral  $\text{Al}^{3+}$  sites is not the same in different regions, as shown in figure 6(a). When the size of the selected region increases, the difference in the number of tetrahedral  $\text{Al}^{3+}$  sites still exists. This will lead to different amounts of

$\text{Cr}^{4+}$  ions doped in different regions in the  $\text{Cr}^{4+}$ :YAG crystal. As shown in figure 6(b), the number of doped  $\text{Cr}^{4+}$  ions in the A and B regions will be different in the  $\text{Cr}^{4+}$ :YAG crystal, which causes different modulation of the laser in the A and B regions. When we select two adjacent regions of the same size ( $20 \mu\text{m} \times 20 \mu\text{m} \times 1.5 \text{ mm}$ ) in the  $\text{Cr}^{4+}$ :YAG crystal, the number of tetrahedral  $\text{Al}^{3+}$  sites in the two regions differs by approximately  $5.8 \times 10^{10}$ , by numerical calculation. This indicates that the amount of doped  $\text{Cr}^{4+}$  ions will be different in different regions of the  $\text{Cr}^{4+}$ :YAG crystal. As shown in figure 6(c), the abscissa indicates the  $r$ -axis in the horizontal direction, and the ordinate  $N(\text{Cr})$  indicates the doping concentration of  $\text{Cr}^{4+}$  ions. The average doping concentration of  $\text{Cr}^{4+}$  ions in region A is higher than that in region B, and the change is periodic along the  $r$  axis, where  $n_0$  indicates the average doping concentration of  $\text{Cr}^{4+}$  ions in the entire  $\text{Cr}^{4+}$ :YAG crystal, and  $d_N$  represents the fluctuation range of the doping concentration of  $\text{Cr}^{4+}$  ions. When the amount of doped  $\text{Cr}^{4+}$  ions is fixed, if the average doping concentration of  $\text{Cr}^{4+}$  ions in one region is high, there would be a region with a lower doping concentration of  $\text{Cr}^{4+}$  ions around this region. As we reduce the size of the selected region, the change in  $\text{Cr}^{4+}$  ion doping concentration along the  $r$ -axis will periodically change continuously, as shown by the red curve in figure 6(c). We define the region with a high concentration of  $\text{Cr}^{4+}$  ions as a  $\text{Cr}^{4+}$ -ions domain, shown in figure 6(c). When the  $\text{Cr}^{4+}$ -ions domain reaches the order of micrometers in size (here the size of the  $\text{Cr}^{4+}$ -ion domain is about  $40 \mu\text{m} \times 40 \mu\text{m} \times 1.5 \text{ mm}$ ), it will have a modulation effect on the formation of the laser mode. Although the difference in  $\text{Cr}^{4+}$  ion concentration among the  $\text{Cr}^{4+}$ -ion domains and surrounding area is small, it is enough to control the loss of different laser modes in the cavity. And the nonlinear saturable absorption of these  $\text{Cr}^{4+}$ -ion domains plays a key role in the formation of IG laser modes in the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip lasers under tilted pumping.

The  $\text{Cr}^{4+}$ -ion domains formed in  $\text{Cr}^{4+}$ :YAG crystal work as a spatial light filter and determine the gain and loss distribution in the intracavity laser. At the same time, the formation of the laser modes is also affected by the pump beam shape and pump area. Besides the shape and incident angle of the





**Figure 7.** The temperature distribution in the cross section of the crystal.

pump beam, the thermal lens effect of the crystal induced by the thermal loading is also affected by the pump area for possible laser mode oscillation. The steady state temperature distribution in the Nd:YAG crystal for an end-pumped Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser can be expressed as [32, 33]

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T(r, z)}{\partial r} \right) + \frac{\partial^2 T(r, z)}{\partial z^2} = -\frac{1}{k_c} q(r, z), \quad (4)$$

where the heat source from the pump power can be described as a super-Gaussian shaped distribution of the pump beam

$$q(r, z) = \frac{2\eta_h P_{in} \alpha}{\pi w_p^2} \exp\left(-\frac{2r^4}{w_p^4}\right) \exp(-\alpha z), \quad (5)$$

where  $\alpha$  is the absorption coefficient of Nd:YAG crystal,  $\eta_h$  is the heat transfer coefficient,  $P_{in}$  is the incident pump power,  $k_c$  is the heat conduction constant,  $w_p$  is the radius of the pump beam.

The spatial temperature distribution inside the Nd:YAG crystal under the single emitter laser diode pumping is calculated according to equation (4). The material parameters of Nd:YAG crystal used in the calculation are  $\alpha = 7.8 \text{ cm}^{-1}$ ,  $\eta_h = 0.32$ ,  $k_c = 0.014 \text{ W (mm} \cdot \text{K)}^{-1}$ ,  $l = 1.8 \text{ mm}$ , and the incident pump power is  $P_{in} = 2.9 \text{ W}$ . Figure 7 shows the temperature distribution in the cross section of the crystal when  $z = 0$ , the highest temperature is about  $60.7 \text{ }^\circ\text{C}$  at the center of the crystal, and the temperature is about  $31.8 \text{ }^\circ\text{C}$  at the edge of the crystal. Since the temperature in Nd:YAG crystal exhibits a gradient change in the horizontal direction ( $x$ ) and the vertical direction ( $y$ ), it inevitably leads to a gradient change for the focal length ( $f_t$ ) of the thermal lens. The  $f_t$  gradually increases from the central area to the edge area along the horizontal direction or the vertical direction. The temperature gradient in the horizontal direction is higher than the temperature gradient in the vertical direction, so the  $f_t$  in the horizontal direction is smaller than the thermal lens focal length in the vertical direction. Therefore, the focus effect of the pump spot in the horizontal direction will be significantly stronger than the focus effect in the vertical direction.

The center axis of the resonator is taken as the  $z$ -axis to establish the Cartesian coordinate ( $x, y, z$ ). The tilted pump beam from the single emitter laser diode can be expressed as

$$E(x, y, z) = E_0 \cdot \exp \left[ - \left( \frac{x^4}{w_x(z \cos(\theta) + y \sin(\theta))^4} + \frac{[-z \sin(\theta) + y \cos(\theta)]^4}{w_y(z \cos(\theta) + y \sin(\theta))^4} \right) \right], \quad (6)$$

where  $\theta$  denotes the incident angle of the pump beam at the rear cavity mirror of this resonator.

When the thermal lens effect of the Nd:YAG crystal is taken into account, the propagation of the pump beam in the cavity is affected. The contraction factors  $\beta_x(y)$  and  $\beta_y(x)$  have been introduced to describe the effect of the thermal lens induced by the pump beam along the horizontal and vertical directions, and  $w_x(z)$  and  $w_y(z)$  can be modified as

$$w_x(z) = \sqrt{w_{0x}^2 \left[ 1 + \frac{\beta_x(y) (M_x^2)^2 \lambda_p^2 z^2}{\pi^2 w_{0x}^4 n^4} \right]}, \quad (7)$$

$$w_y(z) = \sqrt{w_{0y}^2 \left[ 1 + \frac{\beta_y(x) (M_y^2)^2 \lambda_p^2 z^2}{\pi^2 w_{0y}^4 n^4} \right]}, \quad (8)$$

where  $w_{0x}$  and  $w_{0y}$  are the focused pump beam waists along the  $x$ -axis and  $y$ -axis, respectively,  $n$  is the refractivity of the laser crystal,  $M_x^2$  and  $M_y^2$  are the beam quality factors in the  $x$ -direction and the  $y$ -direction, respectively,  $\lambda_p$  is the wavelength of the pump beam. In addition,  $\beta_x(y) \leq 1$  and  $\beta_y(x) \leq 1$ . The thermal lens has no effect on the laser beam when  $\beta_x(y)$  and  $\beta_y(x)$  are equal to one.

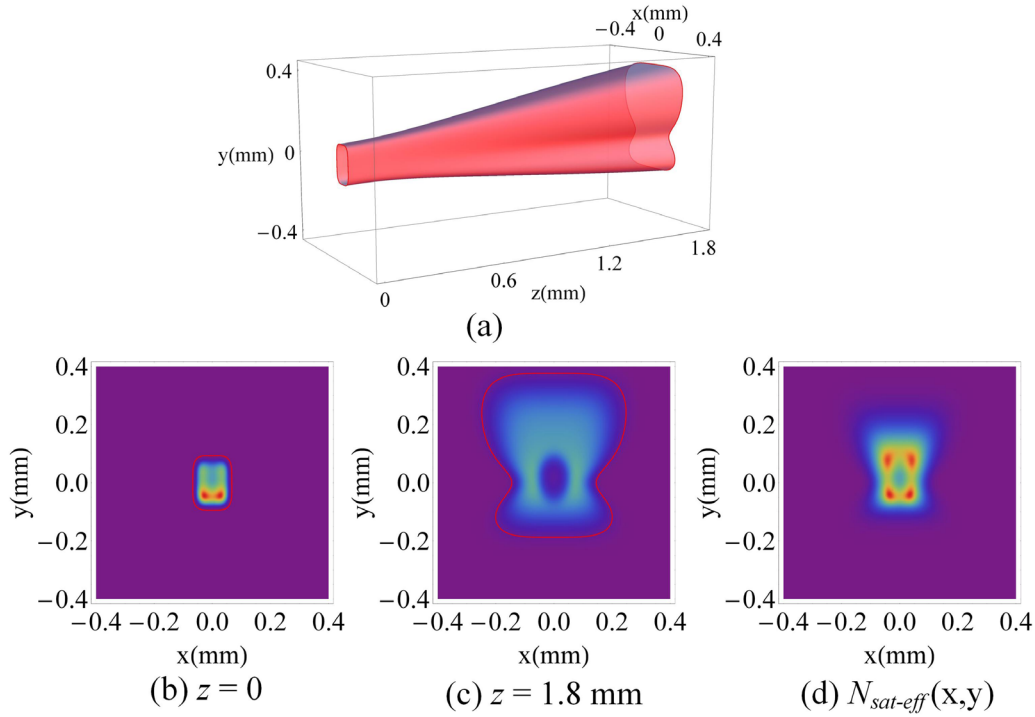
Because the thermal lens effect induced by the pump power does not only affect the pump beam and laser beam propagation inside the gain medium, but also has a great effect on the distribution of the inversion populations, the inversion population distribution inside the Nd:YAG crystal is calculated theoretically by introducing the gradient thermal lens effect along the  $x$  and  $y$  directions. The thermal lens dependent inversion populations can be represented as

$$\Delta N(x, y, z) = \frac{2P_{in} \alpha f_a \tau}{h \nu_p \pi w_x(z_1) w_y(z_1)} \cdot \exp \left[ -2 \left( \frac{x_1^4}{\omega_x(z_1)^4} + \frac{y_1^4}{\omega_y(z_1)^4} \right) \right] \cdot \exp \left[ -\alpha \frac{z}{\cos(\theta)} \right], \quad (9)$$

and

$$\begin{cases} x_1 = x \\ y_1 = -z \sin(\theta) + y \cos(\theta) \\ z_1 = z \cos(\theta) + y \sin(\theta) \end{cases}, \quad (10)$$

where  $f_a$  is the fractional equilibrium Boltzmann population of the upper laser level in the crystal field component (one for a four-level system),  $\tau$  is the fluorescence lifetime of the gain crystal,  $h$  denotes the Planck constant,  $\nu_p$  is the frequency of the pump beam.



**Figure 8.** The distribution of the saturated inversion populations induced by the gradient thermal lens effect along fast-axis and slow-axis of the single-emitter laser diode. (a) The  $N_{\text{sat}} = N_i$  contour surface inside the Nd:YAG crystal; (b) distribution of the saturated inversion population at  $z = 0$ ; (c) distributions of the saturated inversion population at  $z = 1.8$  mm, the red solid lines represents  $N_{\text{sat}} = N_i$  contour lines; (d) Effective saturated inversion population distribution along the thickness of Nd:YAG crystal.

The saturated inversion population under the high laser intensity is given by [22]:

$$N_{\text{sat}}(x, y, z) = \Delta N(x, y, z) / [1 + I(x, y, z) / I_{\text{sat}}], \quad (11)$$

where  $I(x, y, z)$  is the intensity of the laser mode inside the laser resonator and  $I_{\text{sat}}$  is the laser saturation intensity of the laser crystal.

The initial inversion population of the Nd:YAG/Cr<sup>4+</sup>:YAG PQS laser under CW pumping can be written as [22]

$$N_i = [2\sigma_{\text{gs}}N_{\text{s0}}l_s + \ln(1/R_{\text{oc}}) + \delta_{\text{Loss}}] / (2\sigma l), \quad (12)$$

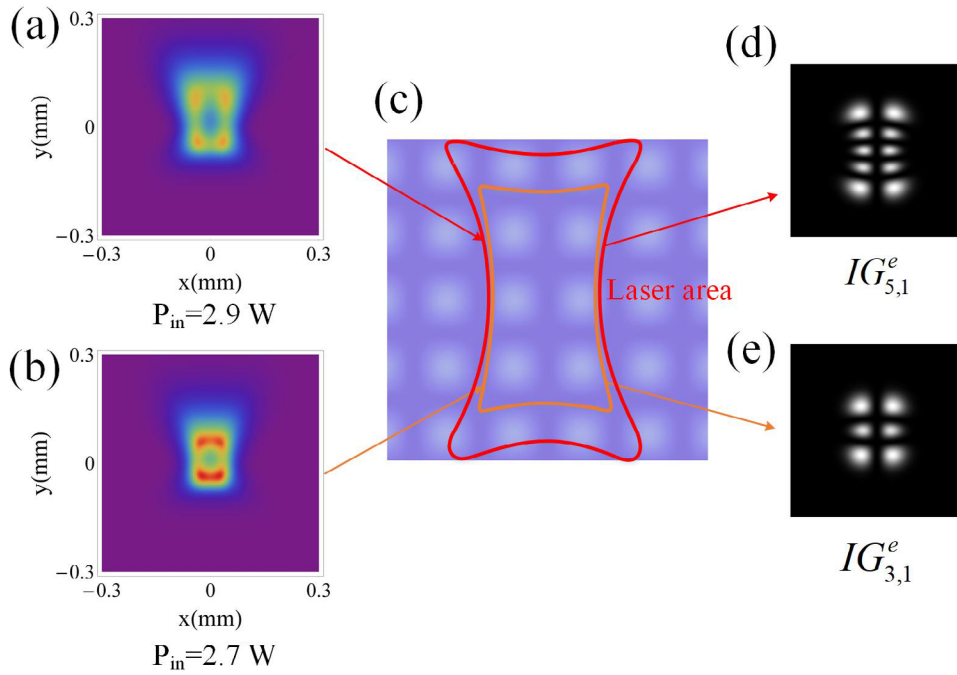
where  $\sigma$  is the emission cross section of gain medium,  $N_{\text{s0}}$  is the total concentration of Cr<sup>4+</sup> in the Cr<sup>4+</sup>:YAG,  $l$  is the length of the Nd:YAG crystal,  $l_s$  is the length of the Cr<sup>4+</sup>:YAG crystal,  $R_{\text{oc}}$  is the reflectivity of the output coupler, and  $\delta_{\text{Loss}}$  is the total intracavity loss.

Figure 8 shows the theoretically calculated saturated inversion population distributions inside the Nd:YAG crystal in the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser under tilted pumping when the gradient thermal lens effect along the  $x$ -direction and the  $y$ -direction are taken into account. The pump beam focus spot is located on the rear surface of the Nd:YAG crystal ( $z = 0$ ). The parameters used in the theoretical calculation are as follows:  $\sigma = 2.8 \times 10^{-19}$  cm<sup>2</sup>,  $\sigma_{\text{gs}} = 4.3 \times 10^{-18}$  cm<sup>2</sup>,  $l = 1.8$  mm,  $l_s = 1.5$  mm,  $N_{\text{s0}} = 4.89 \times 10^{16}$  cm<sup>-3</sup>,  $R_{\text{oc}} = 95\%$ ,  $P_{\text{in}} = 2.9$  W,  $\alpha = 7.8$  cm<sup>-1</sup>,  $\theta = 3^\circ$ ,  $\delta_{\text{Loss}} = 0.1$ ,  $\tau = 230$   $\mu$ s. Figure 8(a) gives the 3D saturated inversion population distribution inside the Nd:YAG crystal when the saturated inversion population is equal to the initial inversion population of the Nd:YAG/Cr<sup>4+</sup>:YAG PQS microchip laser.

The saturated inversion population distribution gives the possible laser mode area inside the resonator. Due to the influence of the thermal lens with gradient focal length, the divergence angle of the pump beam at different positions is different, and the divergence angle of the pump beam at the center position is smaller than the divergence angle of the pump beam at the periphery. The distributions of the saturated inversion populations at  $z = 0$  and  $z = 1.8$  mm are shown in figures 8(b) and (c), respectively. Inside the Nd:YAG crystal, the distribution area of the saturated inversion population gradually increases along the thickness, while the saturated inversion population density gradually decreases owing to the exponential decay of the absorbed pump power. The saturated inversion population distribution on any cross sections along the thickness of Nd:YAG crystal cannot be used to accurately analyze the formation of laser modes. To reflect the overall effect, we define the effective saturated inversion population distribution to represent the average distribution of saturated inversion population in the cavity. When the length of the laser crystal is  $l$ , the effective saturated inversion population is expressed as

$$N_{\text{sat-eff}}(x, y) = \int_0^l N_{\text{sat}}(x, y, z) dz / l. \quad (13)$$

The effective saturated inversion population is calculated, as shown in figure 8(d). The inversion populations in the central region are significantly contracted inward in the horizontal direction, and the contraction amplitude gradually decreases from the center to both sides. This provides a favorable gain region for generating the IG mode.



**Figure 9.** Spatial modulation effect of  $\text{Cr}^{4+}$ -ion domains on selecting IG modes for Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser at different pump powers. (a) Effective saturated inversion population distribution at  $P_{in} = 2.9$  W; (b) effective saturated inversion population distribution at  $P_{in} = 2.7$  W; (c) distribution of the  $\text{Cr}^{4+}$ -ion domains along [1 1 1] direction; (d)  $IG_{5,1}^e$  mode obtained at  $P_{in} = 2.9$  W; (e)  $IG_{3,1}^e$  mode obtained at  $P_{in} = 2.7$  W.

In order to illustrate the spatial selection of IG mode oscillation in the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser by the  $\text{Cr}^{4+}$ -ion domains as a spatial filter,  $IG_{5,1}^e$  mode and  $IG_{3,1}^e$  mode are taken as examples (see figure 2). Figures 9(a) and (b) show the effective saturated inversion population distribution at  $P_{in} = 2.9$  W and 2.7 W, respectively. The effective saturated inversion population density increases with the increase in pump power, and makes possible the laser area increase. The distribution of the  $\text{Cr}^{4+}$ -ions domain along the [1 1 1] direction is shown in figure 9(c). Each bright spot represents a  $\text{Cr}^{4+}$ -ion domain of about  $40 \mu\text{m} \times 40 \mu\text{m}$  and the area surrounded by red curves in figure 9(c) represents the laser oscillation region at  $P_{in} = 2.9$  W. When the  $\text{Cr}^{4+}$ -ions domain is formed in the order of micrometers in size, it will have a significant spatial modulation effect on the formation of the laser mode. There are ten  $\text{Cr}^{4+}$ -ion domains in the laser oscillation region, and these domains have a nonlinear saturated absorption for the intracavity laser, the laser energy is stored in this areas until the saturable absorber is bleached and laser pulses with selected laser mode are generated, and the laser pattern ( $IG_{5,1}^e$ ) shown in figure 9(d) is obtained. When the  $P_{in}$  drops to 2.7 W, the laser oscillation region surrounded by orange curves in figure 9(c) decreases significantly. This is due to the decline in the saturated inversion population. There are only six  $\text{Cr}^{4+}$ -ion domains in the laser oscillation region, and the order of the laser mode is significantly reduced,  $IG_{3,1}^e$  mode is obtained in the PQS microchip laser, as shown in figure 9(e). At low pump power, the pump area is generally larger than the actual laser oscillation region, however, when  $P_{in}$  is high, the laser oscillation region may be equal to or larger than the size of the pump

region due to the gain guidance effect, and the higher order IG mode will be generated.

For the Cr,Nd:YAG SQS microchip laser with  $T_0 = 94\%$ , the  $\text{Cr}^{4+}$  ion concentration was estimated to be  $7.99 \times 10^{16} \text{ cm}^{-3}$ , which is comparable to that of the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser with  $T_0 = 95\%$ . A 1.8 mm-thick Cr,Nd:YAG crystal is longer than the 1.5 mm-thick  $\text{Cr}^{4+}$ :YAG used in the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser. This indicates that for the same amount of Cr ions doped in the Cr,Nd:YAG crystal and  $\text{Cr}^{4+}$ :YAG crystal, the modulation depth and losses introduced in the Cr,Nd:YAG microchip laser is high. Therefore, the order of the IG laser modes generated in the Cr,Nd:YAG SQS microchip laser is lower than that obtained in the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip lasers under the same pump power. Also, the pump power threshold of the Cr,Nd:YAG SQS microchip laser is higher than that of the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser. The performance of the Cr,Nd:YAG SQS microchip laser is degraded compared to that of the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser.

## 5. Conclusion

The formation of IG modes in tilted beam pumped Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS and Cr,Nd:YAG SQS microchip lasers has been investigated experimentally and theoretically. The interaction of the  $\text{Cr}^{4+}$  ion distribution in YAG crystal and the deformed laser beam induced by the thermal lens is attributed to the formation of IG modes in passively Q-switched microchip lasers. The asymmetric saturated inversion population

distributions inside the gain medium are responsible for a platform that forms IG modes with  $\text{Cr}^{4+}$ -ion domains as the spatial light filter. The inversion population is selectively saturated by the  $\text{Cr}^{4+}$ -ion domains when the laser intensity is high enough, which makes the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser oscillate in IG modes. The work shows a clear image of IG mode oscillation in the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser under tilted beam pumping. The controllable IG modes could be obtained by carefully selecting the pump beam distribution and modulation depth of  $\text{Cr}^{4+}$ :YAG crystal in the Nd:YAG/ $\text{Cr}^{4+}$ :YAG PQS microchip laser.

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