Surface loss limit of the power scaling of a thin-disk laser

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We show that the general limit of power scaling of thin-disk lasers comes not only from the overheating and amplified spontaneous emission (ASE) but also from the surface loss. Overheating or thermal fracture increases the transverse size at the scaling, whereas ASE limits the gain-size product. The gain coefficient should decrease at the scaling. However, the round-trip gain should remain larger than the background loss; hence, the thickness should increase at the scaling. The limit of the output power per single active element occurs when the medium becomes too thick and cannot work efficiently without overheating. The maximum output power scales inversely with the cube of the surface loss coefficient. In the quasi-continuous regime, the average power scales inversely to the product of the duration of pulses to the repetition rate. © 2006 Optical Society of America

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1. INTRODUCTION

Thin disk lasers are under intensive study (see 1–10 and references therein). The thin-disk geometry is used for a broad range of output powers, from microchips[7] to the nuclear fusion driver.[3,10] Thin-disk geometry is characterized by the large aspect ratio of the lateral size to its length (Fig. 1), so the medium appears as a thin disk.[11] The main advantages of disk lasers are the efficient cooling and the possibility of reducing thermal lensing, so that power scaling with good beam quality is possible. Impressive achievements in the technology of thin-disk lasers should be mentioned. Efficient operation at the kilowatt level using a 3 mm diameter active element has been demonstrated.[9] The 2 kW laser, using the thin-disk geometry with a multipass pumping configuration, is already available commercially.[12] The ceramics technology allows the fabrication of slabs of active medium of size of order of 1 m (see Ref. 12); so, a power scaling to much higher level seems easily achievable, and the proposals for the 100 kw laser have been patented.[13] The successive scaling of power to values necessary for the laser fusion driver is expected for the near future.

The scaling limits of thin-disk lasers have already been considered.[14–16] In this paper, we analyze the influence of the surface loss and show that the latter plays a key role in limiting the maximum achievable power. We show that the combined effect of overheating, amplified spontaneous emission (ASE), and the surface-scattering loss limit the power scaling. Although the loss at the surface is small and usually neglected in the power-scaling analyses published so far, we show that the maximum achievable power is very sensitive to the scattering-loss coefficient. Even at the power 1 order of magnitude lower than the maximal value, our model describes the drop of the efficiency due to the surface-scattering loss.

In this paper, we do not consider the storage of energy in the active medium, although such a storage is essential for some applications (for example, the nuclear fusion driver). There are scaling laws for this storage too; this will be a subject of a separate paper.

2. MODEL OF LASER AND ASE

Consider a slab of some active medium placed on a heat sink as shown in Fig. 1. Let h be its thickness and L be its transverse size. For simplicity, assume that the pumped part of the active medium is a block $L \times L \times h$. Let G be the gain and $g = 2Gh$ be the round-trip gain. Let β be the coefficient of background loss of the signal per active element. This loss may be caused by the scattering.

Assume that a good medium is chosen that allows the efficient laser action. Assume that it is uniformly pumped (perhaps, with powerful laser diodes); assume that the intensities of the pump and of the signal are almost constant. Assume that the surface background loss coefficient $\beta \ll 1$ is determined by technological reasons. What are required values of g, h and L to reach the maximal power? What is the power limit? To what efficiency does it correspond?

To simplify the model, we neglect all physical effects that are not essential for the limit of the scaling and deduce the simple analytical estimate for the parameters mentioned. Here we consider only cw operation, although the scaling laws for the storage of energy in an active medium can be considered in similar manner.

We assume efficient delivery of pump. Some kind of pump trapping, multipass scheme,[9] and/or transversal delivery[17] can be used. We suppose that the ideal flat mirror is placed between this slab and the heat sink. The possible loss at this mirror can be included into the coupling loss β. We assume low thermal resistance of the heat sink, so the bottom of the device is kept at the constant temperature.

We try to keep our consideration as general as possible. Therefore we do not show in Fig. 1 details that may depend on the specific design of the disk laser. We do not

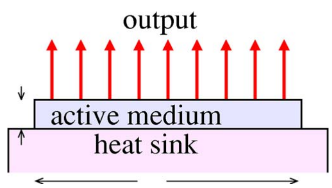

Fig. 1. (Color online) Thin-disk laser as an active mirror. Pump is assumed to be efficiently absorbed in the medium, which may require multipass and/or lateral delivery.

show the absorber of ASE around the disk, which is important to avoid the parasitic oscillations. (Such an absorber may convert most of the power of the ASE into heat; however, it may also convert some part of the ASE into light of lower frequency.)

We do not show the output coupler. The design of the output coupler may affect the ASE. Some photons leave from the active medium at the first passing through the upper surface, especially if an external output coupler is used. Some of photons have multiple internal reflections. Such a photon, traveling from one corner of the rectangular slab to the opposite corner, may get the total length of the order of $2L$ in the active medium. However, the high-reflection coating of the output coupler has no need to provide the good reflection of the ASE coming at some angles significantly different from that of the signal; so we expect the position of the output coupler is not crucial for the mean path of the ASE in the active medium. Following Ref. 6, we assume that the average photon of the spontaneous emission travels an average distance L. The averaged photon of spontaneous emission becomes $\exp(GL)$ photons as it leaves the active medium. This increases the effective relaxation rate of the medium by a factor $\exp(GL)$. Let τ_o be the lifetime of the upper laser manifold. The effective lifetime can be estimated as

$$\tau = \tau_o \exp(-GL). \tag{1}$$

However, the average length that the spontaneous photon passes through the active medium may be reduced if we merge the disk with a passive medium with a matched refractive index. Also, the design of the output coupler may affect the path of a photon of the spontaneous emission in the medium. Here we consider the simplest configuration. We do not take into account the spatial distribution of pump and the signal intensity. This distribution is not essential for the limit of the size of the device. Therefore we assume that the signal profile is flat-top, and treat all intensities as constants.

We assume that the device (Fig. 1) is a laser oscillator. However, a similar estimate can also be applied to the power amplifier; then, the power increment per single active element required becomes an equivalent of the output coupling parameter θ; and the estimate of the maximal signal power of oscillator becomes a maximal signal power per active element in the cascade of power amplifiers.

3. HEAT AND THICKNESS

One of the factors limiting the size of lasers is the overheating. Assume that the thickness h is small compared with the width L of the pumped region. Then the heat flow is almost one dimensional. Assuming the constant thermal conductivity k of the medium and uniform heat generation, the heat diffusion equation leads to the simple estimate for the difference ΔT of temperature from the heat sink to the upper surface, (where the thermal flux is assumed to be zero):

$$\Delta T = \frac{\eta_h P_p h}{2kL^2}, \tag{2}$$

where P_p is the absorbed pump power, $\eta_h P_p$ is power converted to heat, and η_h is the heat generation parameter. For the efficient operation this parameter can be estimated as

$$\eta_h = \frac{\omega_p - \omega_s}{\omega_p} = 1 - \eta_o, \tag{3}$$

where η_o is the ratio of frequency ω_s of signal to the frequency ω_p of the pump:

$$\eta_o = \omega_s / \omega_p. \tag{4}$$

However, the actual heat generation is slightly higher than that the estimate [Eq. (3)] assumes.[18]

From Eq. (2), we obtain the estimate for the maximal pump power that can be delivered to the laser medium

$$P_{p,\max} = \frac{2k \Delta T_{\max} L^2}{\eta_h h}, \tag{5}$$

where $\Delta T_{\max}$ is maximal increment of the temperature the medium can get without deteriorating its performance.

The pump power may also be limited by the fracture of the disk material. This limit can be expressed in terms of the thermal shock parameter[19]

$$R_T = \frac{k \sigma_T (1-\nu)}{\alpha E}, \tag{6}$$

where σ_T is the maximal tension allowed in the medium, ν is the Poisson ratio, E is the Young modulus, and α is the thermal expansion coefficient. Then, the limit of the pump power can be estimated as

$$P_{p,\max} = \frac{3R_T L^2}{\eta_h h}. \tag{7}$$

This formula derives directly from the estimate (4.18) by Ref. 1 of stress of active slab. (Our $2h$ is equivalent to the parameter d from Ref. 1).

For four-level systems, such as Nd-doped materials, fracture limit determines the maximum power, whereas for quasi-four-level systems, such as Yb-doped media, overheating is the dominant factor. The maximum power is limited by the lower of the two estimates (5) and (7). Combining these expressions, we get the estimate

$$P_{p,\max} = \frac{RL^2}{h}, \tag{8}$$

where the thermal loading parameter is

$$R = \min \begin{cases} 3R_{\mathrm{T}}/\eta_{\mathrm{h}} \\ 2k\Delta T/\eta_{\mathrm{h}} \end{cases}. \tag{9}$$

We assume that ASE is not dissipated inside the active medium; it should be either recycled as source for pumping some other medium with lower working frequency or absorbed in some ring around the disk (not shown in Fig. 1).

At fixed frequencies ω_{p}, ω_{s} of pump and signal, the parameter R by Eq. (9) appears as property of the medium. The thermal conductivity k may be of order of several W/(m·K), depending on the gain medium and the temperature of operation.[3] Typically, in order to keep the performance of the medium, the temperature should not increase more than few hundred Kelvins.[20] The heat generation parameter η_{h} in Eq. (9) can be of order 10%; this gives an additional factor of the order of ten. In the following examples, we use value $R=5$ W/mm. However, the update to other values is straightforward.

4. OUTPUT COUPLING LOSS

The round-trip gain $g=2Gh$ compensates both the round-trip loss and the output coupling in such a way that

$$(1-\theta-\beta)\exp(g) = 1, \tag{10}$$

where β is the loss parameter that includes the scattering of the signal at the surface.

However, the background absorption in this disk elements, usually of the order of 0.2 m^{-1}, makes a negligible contribution compared with the surface scattering loss. For example, the background loss of a 1 mm-thick disk becomes comparable with the surface loss when the latter is about 10^{-5}. Although such low loss is achievable,[21] it is not yet available for the powerful disk lasers; values of around $\beta=1\%$ look more reasonable for estimates. In this paper, we ignore volumetric scattering. However, we assume that $\beta < g \ll 1$; then Eq. (10) implies that $g=\theta+\beta$.

Consider the ratio of the power the signal carries out to the total power the medium transfers to the signal. We interpret this ratio as efficiency of the output coupling of the signal. Per round trip, the signal gets power $gI_{\mathrm{s}}L^2$, where I_{s} is the signal intensity inside the medium. The output coupler releases power $\theta I_{\mathrm{s}}L^2$. The surface scatter power is $\beta I_{\mathrm{s}}L^2$. The efficiency of output coupling can be estimated as the ratio of these two powers:

$$\eta_{\mathrm{output}} = \frac{\theta I_{\mathrm{s}}L^2}{gI_{\mathrm{s}}L^2} = 1 - \frac{\beta}{g}. \tag{11}$$

For efficient extraction of power, the coefficient β should be small compared with round-trip gain g.

To minimize the number of parameters, we use only one parameter $g=\beta+\theta$ in order to characterize the coupling efficiency. Then, the gain is determined by

$$G = g/(2h) = (\beta+\theta)/(2h). \tag{12}$$

In this section, we have determined gain in terms of the properties of the output coupler, and corresponding efficiency η_{output} of the output coupling.

5. SATURATION PARAMETER AND THE THRESHOLD

The threshold of generation can be analyzed using a simple model for the active medium.[17] In Appendix A, we supply the summary necessary for such an estimate. A similar estimate can be obtained also using Eq. (1.36) of Ref. 22 by scaling Eq. (1) of the relaxation time, so the deduction is moved into Appendix B.

We assume that the scheme of delivery of pump is already optimized, according to the optimized concentration of the active medium; such an optical concentration is estimated in Appendix B. To keep the consideration as general as possible, we should reduce the number of parameters characterizing the medium. Define

$$Q = \frac{\hbar\omega_{\mathrm{p}}}{2\tau_0(\sigma_{\mathrm{se}} - \sigma_{\mathrm{sa}}\sigma_{\mathrm{pe}}/\sigma_{\mathrm{pa}})} \approx \frac{\hbar\omega_{\mathrm{p}}}{2\tau_0\sigma_{\mathrm{se}}}, \tag{13}$$

where σ_{se} and σ_{sa} are the effective emission and absorption cross-sections of the active medium at the frequency ω_{s} of the signal and σ_{se} and σ_{sa} are the same at the frequency ω_{p} of the pump. Usually, $\sigma_{\mathrm{se}}\sigma_{\mathrm{pa}} \gg \sigma_{\mathrm{sa}}\sigma_{\mathrm{pe}}$, so, we can use the expression in the right- hand side of Eq. (13). Parameter Q can be interpreted as some effective saturation intensity. At fixed ω_{p} and ω_{s}, the saturation parameter Q appears as property of the medium. With this parameter, the threshold can be expressed (see Appendix B) as

$$P_{\mathrm{th}} = QL^2 g e^{GL}. \tag{14}$$

If we remove the exponential factor, this estimate becomes a special case of Eq. (1.36) from Ref. 22 for the pump profile, perfectly matching that of profile of signal, so the estimate [Eq.(14)] can be obtained from Eq. (1.36) of Ref. 22 by scaling the relaxation time [Eq. (1)] due to the ASE.

6. OUTPUT POWER

We suppose that the frequency ω_{p} of pump and the frequency ω_{s} of the signal are fixed. Then we characterize the medium with parameters η_{o}, R and Q. The surface-loss parameter β also can be interpreted as the given property of the active material. The design of a thin disk is specified with the size L, thickness h (Fig. 1), and the round-trip gain $g=\theta-\beta$. In this section, we estimate the output power and corresponding efficiency in terms of these parameters.

The output signal power can be expressed as

$$P_{\mathrm{s}} = \eta_{\mathrm{o}}(1-\beta/g)(P_{\mathrm{p}} - P_{\mathrm{th}}), \tag{15}$$

where P_{p} is the input pump power. This is a linear function shown in Fig. 2a. In our model, the conversion efficiency

$$\eta = P_{\mathrm{s}}/P_{\mathrm{p}} \tag{16}$$

increases the function of power [Fig. 2b]. The specific curves for the efficiency can be found in Ref. 20.

Consider the output power at maximum pump power allowed at given configuration. We already have an estimate [Eq. (14)] for the threshold power P_{th} and estimate [Eq. (8)] for the maximal power $P_{\mathrm{p,max}}$. The maximal

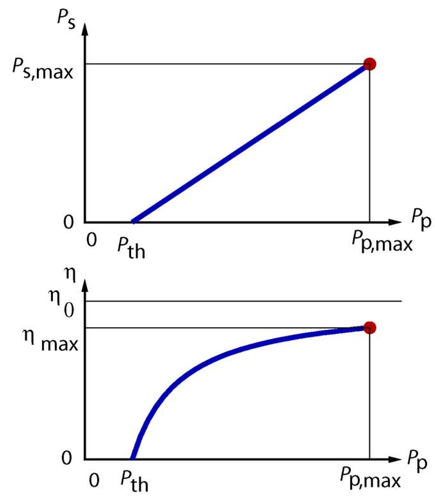

Fig. 2. (Color online) Output power P_s and efficiency $\eta = P_s/P_p$ versus pump power.

power pump power P_p is limited by the heat load parameter R by Eq. (9). The output power depends on the saturation parameter Q by Eq. (13). We may optimize the output power P_s estimated by Eq. (15) with respect to parameters g, h, and L.

The substitution of Eq. (14) into Eq. (15) gives the equation

$$P_s = \eta_o (1 - \beta/g)(P_p - QL^2 g e^u), \tag{17}$$

where

$$u = GL = \frac{gL}{2g}, \quad P_p = RL^2/h. \tag{18}$$

At the maximal output power, the derivatives of Eq. (17) with respect to h and L and round-trip g should be zero; these equations determine the optimal values of parameters h, L, and g. Unfortunately, such equations are transcendental, and it is difficult to express the optimal values in terms of the elementary functions.

The optimization of output power becomes straightforward, if we use g, P_p, and u as new independent variables. Then, from the condition (18), we express

$$L = \frac{P_p g}{2uR}, \quad h = \frac{P_p g^2}{4u^2 R} \tag{19}$$

and rewrite the estimate [Eq. (17)] in terms of $\{g, P_p, u\}$ as follows:

$$P_s = \eta_o \left(1 - \frac{\beta}{g}\right)\left(P_p - Q \frac{P^2 g^3}{4R^2} \frac{e^u}{u^2}\right). \tag{20}$$

The only expression e^u/u^2 carries the dependence on u; the maximum output corresponds to $u=2$. This gives

$$P_s = \eta_o \left(1 - \frac{\beta}{g}\right)\left(P_p - \frac{e^2}{16} \frac{Q}{R^2} g^3 P_p^2\right). \tag{21}$$

The optimization of parameter g depends on the aim, should we just get the maximal power or we need to also achieve the efficient operation.

7. MAXIMUM OUTPUT POWER

In this section, we find the maximum of Eq. (21) with respect to the pump power P_p and round-trip gain g, regardless the resulting efficiency. Setting to zero the derivative with respect to P_p, we get

$$P_p = \frac{8R^2}{e^2 Q g^3}. \tag{22}$$

The substitution of Eq. (22) into Eq. (21) gives the estimate of the maximal output power achievable at given round-trip gain g:

$$P_s = \eta_o \left(1 - \frac{\beta}{g}\right) \frac{4R^4}{e^2 Q g^3}. \tag{23}$$

The maximization with respect to g gives

$$g = \frac{4}{3}\beta. \tag{24}$$

Substituting Eq. (24) into Eq. (22) we get the estimate of the optimum pump power

$$P_p = P_{p,max} = \frac{27}{8e^2} \frac{R^2}{Q} \frac{1}{\beta^3} \approx 0.457 \frac{R^2}{Q\beta^3}. \tag{25}$$

Using Eq. (23), we get the estimate for the maximum signal output power

$$P_s = P_{s,max} = \frac{27}{64e^2} \frac{R^2}{Q} \frac{\eta_o}{\beta^3} \approx 0.057 \frac{R^2 \eta_o}{Q\beta^3}, \tag{26}$$

which corresponds to the efficiency

$$\eta = \frac{P_{s,max}}{P_{p,max}} = \frac{\eta_o}{8}. \tag{27}$$

From Eq. (19) and condition $u=2$, we recover the optimal values of the size L and the thickness h of the slab:

$$L = \frac{9}{8e^2} \frac{R}{Q} \frac{1}{\beta^2}, \quad h = \frac{3}{8e^2} \frac{R}{Q} \frac{1}{\beta}. \tag{28}$$

The estimate [expression (26)] reveals the importance of the surface loss β. This loss determines the limit of the power scaling.

8. DIMENSIONAL ANALYSIS AND KEY PARAMETER

From expression (26), we see that

$$P_k = \frac{R^2 \eta_o}{Q\beta^3} \tag{29}$$

is a key parameter in the choice of the active material for the power scaling. Its structure can be guessed also from the dimensional analysis. Parameters Q and R are the only dimensional constants, which characterize the medium in our model. Their dimensions are in W/m and W/m^2. Only the combination R^2/Q has dimension of power. Hence, namely this combination should determine the maximal power of the device. The proportionality of the key parameter to the ratio η_o of the energy of photon of signal to that of pump we also could guess without making any deduction or optimization.

Typically, the parameter R^2/Q has moderate values of order of hundred milliwatts. We have no large dimensionless parameters in our model, and only one small parameter, surface loss β. We might expect the output power to be proportional also to some negative power of this small parameter. This leads to Eq. (29).

Similarly, the ratio R/Q is the only combination of the medium parameters which has the dimension of length; so we could guess also the structure of the estimates in Eqs. (28); ratio h/L is proportional to β. In such a way, the structure of estimates of Section 7 is predetermined by the dimensions of the constants we use in the model.

The anti-ASE cap[23] can be used to increase the limit of power and/or the efficiency of a disk laser. In this case the undoped layer prevents the multiple total internal reflections of ASE in the active medium. With such a cap, the disk laser may work with parameter $u=GL$ of an order of 6, and power of ASE will be still comparable with the power of spontaneous emission. This may allow us to scale up the size L with coefficient of an order of 3, giving an additional order of magnitude the output power. While the anti-ASE cap brings no new dimensional constants into the model, the maximum output power of a disk with anti-ASE cap should have a similar scaling law.

We expect our estimates to catch the most important limit of the power scaling of disk lasers. Our estimates do not bring any new effects, which were not taken into account with detailed numerical simulations. However, we expect our simple estimates to reveal the tendency that is common for all kinds of solid-state lasers with a thin active layer (perhaps, including vertical-cavity surface-emitting lasers,[23]) as far as they can be characterized with saturation intensity Q and thermal loading parameter R.

9. EFFICIENT OPERATION

For efficient operation, the pump power should be smaller than its maximal value by Eq. (26). In this section we estimate the maximal output power achievable a given efficiency. Instead of maximization of Eq. (21) with respect to P_p, as we did in Section 6, consider the optimization with respect to g of the efficiency

$$\eta = \frac{P_s}{P_p} = \eta_o\left(1 - \frac{\beta}{g}\right)\left(1 - \frac{e^2}{16}\frac{QP_p g^3}{R^2}\right) \tag{30}$$

at fixed value of P_p. Example of values of maximal power corresponding to various values of the efficiency is presented in Table 1. Corresponding values of L and h are determined by the condition $u=2$ and Eqs. (19). The ratio $P_p/P_{p,\max}$ as function of required efficiency η is presented in the first and last columns of the table. These columns are not specific for the medium; they do not depend on Q, R, and η_o and have a general meaning. For efficient operation, the operating power should be much smaller than the maximal value available for given Q, R, and β.

For the quick estimates, the approximate analytic expression can be more useful than a table. To get such an estimates in compact form, consider the most practical case of the efficient operation. Then each of expressions in parentheses in right hand side of Eq. (30) should be close to unity, and we rewrite our estimate as

Table 1. Scaling of Output Power P_s and Efficiency η of a Disk Laser[a]

$P_p/P_{p\max}$	P_p (kW)	g	L (mm)	h (mm)	P_s (kW)	η
0.00001	0.002	0.201	0.023	0.001	0.002	0.850
0.00010	0.023	0.114	0.130	0.004	0.018	0.804
0.00100	0.228	0.065	0.740	0.012	0.166	0.725
0.01000	2.284	0.037	4.253	0.040	1.355	0.593
0.10000	22.838	0.022	24.937	0.136	8.791	0.385
1.00000	228.378	0.013	152.252	0.508	25.978	0.114

[a] $\beta=0.01$, $R=5$ W/mm, $Q=50$ W/mm^2, and $\eta_o=0.91$.

$$\eta = \eta_o\left(1 - \frac{\beta}{g} - \frac{e^2}{16}\frac{QP_p}{R^2}g^3\right), \tag{31}$$

where we neglect the cross-term $e^2\eta_o\beta QP_p^2 g^2/(16R^2)$. Maximizing Eq. (31) with respect to g, we find the optimum value for the round-trip gain

$$g = \left(\frac{16\beta}{3e^2}\frac{R^2}{QP_p}\right)^{1/4}. \tag{32}$$

Substitution of Eq. (32) into Eq. (31) leads to the estimate for the efficiency optimized for the given pump power P_p:

$$\eta = \eta_o\left[1 - \left(\frac{16e^2QP_p\beta^3}{27R^2}\right)^{1/4}\right]. \tag{33}$$

The optimal values of L and h can be found from Eqs. (32) and (19) at $u=2$. Equation (33) can be inverted to estimate the pump power that corresponds to the given efficiency $\eta=P_s/P_p$:

$$P_p = \frac{27}{16e^2}\left(1 - \frac{\eta}{\eta_o}\right)^4\frac{R^2}{Q\beta^3} \approx 0.228\left(1 - \frac{\eta}{\eta_o}\right)^4\frac{R^2}{Q\beta^3}. \tag{34}$$

Then, the maximum output power at given efficiency η can be estimated as

$$P_s = \eta P_p. \tag{35}$$

As we expect after Section 8, this power is also proportional to the ratio R^2/Q and inversely proportional to the cube of the surface-scattering-loss coefficient β. For high efficiency, the laser should be optimized for the operation at the power of an order of a percent of the maximal power achievable at the given configuration.

Our estimates apply to the single-disk element. From Table 1 we estimate that the realized[8] disk lasers work at the beginning of the drop of the efficiency, owing to the effects we have analyzed. This may be a reason for splitting of the laser system to the set of thin-disk active elements with common cavity or common seed.

10. PULSED REGIME

The effective threshold can be reduced using the pulsed operation. Consider the quasi-cw regime, when the duration Δt of pulses is large compared with the effective re-

Table 2. Latin Notations and Basic Formulas

A	Absorption at the pump frequency [Eq.(A11)]
A_0	Absorption at strong signal [Eq. (A7)]
ASE	Amplified spontaneous emission
$b=A_0/G_0$	Absorption-to-gain ratio [Eq. (B3)]
E	Young modulus [Eq. (6)]
$e=\exp(1)\approx 2.71\ldots$	Base of natural logarithms [Eq. (B9)]
f	Repetition rate in pulsed regime [Eq. (36)]
$G=g/(2h)$	Gain [Eqs. (12) and(A12)]
$g=2Gh=\beta+\theta$	Round-trip gain [Eq. (12)]
h	Thickness of the disk (Fig. 1)
$\hbar\approx 1.05\times 10^{-34}\,\mathrm{J\cdot sec}$	Planck constant [Eq. (B9)]
I_p, I_s	Pump and signal intensities [Eq. (A3)]
$I_\mathrm{po}=\frac{\hbar\omega_\mathrm{p}/\tau}{\sigma_\mathrm{pa}+\sigma_\mathrm{pa}}$	Pump saturation intensity [Eq. (A9)]
$I_\mathrm{so}=\frac{\hbar\omega_\mathrm{s}/\tau}{\sigma_\mathrm{pa}+\sigma_\mathrm{pa}}$	Signal saturation intensity [Eq. (A9)]
$j_\mathrm{p}=I_\mathrm{p}/(\hbar\omega_\mathrm{p})$	Pump flux [Eq. (A3)]
$j_\mathrm{s}=I_\mathrm{s}/(\hbar\omega_\mathrm{s})$	Signal flux saturation [Eq. (A3)]
K	1 deg Kelvin
k	Thermal conductivity [Eq. (2)]
L	Size of the disk (Fig. 1)
$N=G_0/\sigma_\mathrm{e}$	Concentration of active centers [Eq. (A7)]
n_1, n_2	Relative populations of laser manifolds [Eqs. (A4) and (A5)]
$P_\mathrm{k}=R^2\eta_\mathrm{o}/(Q\beta^3)$	Key parameter [Eq. (29)]
P_p	Pump power [Eq. (15)]
P_s	Signal power [Eq. (15)]
$P_\mathrm{th}=\frac{\hbar\omega_\mathrm{p}L^2}{2\sigma_\mathrm{se}\tau_\mathrm{o}}ge^{GL}$	Threshold pump power [Eqs. 22 and(B9)]
$P_\mathrm{s}=\eta_\mathrm{o}(1-\beta/g)(P-P_\mathrm{th})$	Signal power [Eq. (15)]
$p=I_\mathrm{p}/I_\mathrm{po}$	Normalized pump intensity [Eq. (A13)]
$Q=\hbar\omega_\mathrm{p}/(2\tau_\mathrm{o}\sigma_\mathrm{se})$	Saturation parameter [Eq. (13)]
$R=\min\begin{cases}3R_\mathrm{T}/(\eta_\mathrm{h})\\2k\Delta T_\mathrm{max}/(\eta_\mathrm{h})\end{cases}$	Thermal loading [Eq. (9)]
$R_\mathrm{T}=\frac{k\sigma_\mathrm{T}(1-\nu)}{(\alpha E)}$	Thermal shock parameter[19] [Eq. (6)]
$s=I_\mathrm{s}/I_\mathrm{so}$	Normalized signal intensity [Eq. (A13)]
$u=GL$	Mean path gain of ASE [Eq. (18)]
$U=\frac{(\sigma_\mathrm{sa}+\sigma_\mathrm{se})\sigma_\mathrm{pa}}{\sigma_\mathrm{pa}\sigma_\mathrm{se}-\sigma_\mathrm{sa}\sigma_\mathrm{pe}}$	Combination of cross sections [Eq. (A10)]
$V=U-1$	Combination of cross sections [Eq. (A10)]
$X=N\sigma_\mathrm{e}-G=G_0-G$	Displaced gain [Eq. (B6)]

laxation time τ of the medium, and the repetition rate f is still high compared with the rate of decay of variation of temperature. Then, the effective saturation parameter Q by Eq. (13) should be scaled down with factor $f\Delta t$. Although the pulsed operation changes only the threshold without affecting the output coupling efficiency, the scaling down of Q allows the designer to scale up the maximum pumping power, by scaling the thickness h and lateral size L of the device, according to the estimates [Eqs. (28)]. In such a way, we replace $Q\to f\Delta tQ$ in Eq. (35); then Eq. (34) gives the estimate of the maximum mean output power achievable at given efficiency η:

Table 3. Greek Notations and Basic Formulas

α	Thermal expansion coefficient [Eq. (6)]
β	Surface loss coefficient [Eq. (10)]
$\Delta T=\frac{(1-\eta_\mathrm{o})P_\mathrm{p}h}{(2kL^2)}$	Variation of temperature [Eq. (2)]
Δt	Duration of the pump pulse [Eq. (36)]
$\eta=P_\mathrm{s}/P_\mathrm{p}$	Conversion efficiency [Eq. (16)]
η_h	Heat generation parameter [Eq. (2)]
$\eta_\mathrm{o}=\omega_\mathrm{s}/\omega_\mathrm{p}$	Quantum limit of efficiency [Eq. (4)]
$\eta_\mathrm{output}=1-\beta/g$	Efficiency of output coupling [Eq. (11)]
μ	Launching loss of pump [Eq. (B1)]
ν	Poisson ratio [Eq. (6)]
$\theta=g-\beta$	Output coupling [Eq. (10)]
$\tau=\tau_\mathrm{o}\exp(-GL)$	Effective lifetime [Eq. (1)]
τ_o	Lifetime of the upper manifold [Eq. (1)]
$\sigma_\mathrm{a}=\frac{\sigma_\mathrm{pa}\sigma_\mathrm{se}-\sigma_\mathrm{pe}\sigma_\mathrm{sa}}{\sigma_\mathrm{sa}+\sigma_\mathrm{se}}$	Absorption cross section [Eq. (A6)]
$\sigma_\mathrm{e}=\frac{\sigma_\mathrm{pa}\sigma_\mathrm{se}-\sigma_\mathrm{pe}\sigma_\mathrm{sa}}{\sigma_\mathrm{pa}+\sigma_\mathrm{pe}}$	Emission cross section [Eq. (A6)]
σ_pa	Absorption cross section of pump [Eq. (A6)]
σ_pe	Emission cross section of pump [Eq. (A6)]
σ_sa	Absorption cross section of signal [Eq. (A6)]
σ_se	Emission cross section of signal [Eq. (A6)]
σ_T	Maximal tension allowed [Eq. (6)]
ω_p, ω_s	Frequencies of pump and signal [Eq. (4)]

$$P_\mathrm{s}=\frac{27\eta}{16e^2}\left(1-\frac{\eta}{\eta_\mathrm{o}}\right)^4\frac{R^2}{f\Delta tQ\beta^3}. \tag{36}$$

For example, for a repetition rate of $f=10\,\mathrm{Hz}$ and a pulse duration of $\Delta t=10\,\mathrm{ms}$, we can get the output power 1 order of magnitude larger than that in cw regime, at the appropriate scaling up the sizes L and h of the slab.

The quasi-cw operation may increase the limit of the time-average power. Equation (36) shows that the maximum pulse-peak intensity scales inversely with the square of product $f\Delta t$. However, the efficient operation of a laser, optimized for a pulsed regime, is possible only in the pulsed regime; in this case, the laser is too thick and, in the cw operation, it cannot be pumped well without overheating.

Our estimates can be applied also to the laser with short wide unstable cavity.[17] In particular, the examples suggested with high loss at the surface ($\beta=0.05$) have a narrow range of parameters between threshold and overheating. For the robust and efficient device, either the reflectivity should be increased, for example, until 0.99, with corresponding reduction of magnification, or the quasi-continuous regime of operation is required.

11. CONCLUSIONS AND DISCUSSION

The general limit of the power scaling of the disk lasers (Fig. 1) comes from the consideration of surface loss, ASE, the threshold of generation, and the overheating. The ASE limits the gain G, giving the upper bound for the GL product. The surface loss determines the lower limit for the round-trip gain $2Gh$. Therefore the thickness should increase at the scaling. At some critical power, the medium becomes too thick and cannot be pumped well above the threshold without overheating. This limits the power scaling of laser disks. As we approach this limit, the efficiency drops down.

We have considered the simple model that still indicates the limits of the power scaling. We collect our notations and basic formulas in Tables 2 and 3. Our model includes only four parameters: ratio η_o of frequencies of signal and pump, surface loss coefficient β, thermal loading parameter R from Eq. (9), and the saturation parameter Q from Eq. (13).

Our estimates indicate that $R^2\eta_o/(Q\beta^3)$ is a key parameter in the choice of the active medium for the power scaling. This parameter determines the maximum output power $P_{s,max}$ by Eq. (26) per single active mirror. To our knowledge, Eq. (26) is the most general limit of the power of thin-disk lasers. The strong dependence of $P_{s,max}$ on the surface loss is counterintuitive but important in the design of powerful devices.

Within our model, the maximal output power $P_{s,max}$ takes place at a low efficiency of an order of 10%. The efficient laser should be designed for the output power $P_s \ll P_{s,max}$ (Table 1); the drop of the efficiency begins far before the power limit. This explains the use of multiple disks.[2,8,9] The reduction of the surface-scattering loss β of the disk lasers and amplifiers is a way to increase the power per single element of an efficient thin-disk laser.

The average output power can be increased in the quasi-cw pulsed regime. At the duration Δt and repetition rate f, the effective saturation parameter Q scales down with coefficient $f\Delta t$, leading to the estimate in Eq. (36). Then the maximal peak power scales inversely with the square of the $f\Delta t$ product. In particular, examples considered in Ref. 17 for the laser with a short wide unstable cavity are at the edge of overheating; either the loss at the reflection should be reduced or such a laser should operate in quasi-cw regime.

The method used can be extended. Similar estimates can be done for the disk with an anti-ASE cap,[23] the reduction of the threshold may give an additional order of magnitude to the output power. Also, the storage of energy in pulsed laser can be considered in the same approximation. Such researches are matter for the future work.

APPENDIX A: EXAMPLE OF MEDIUM

In this appendix, we suggest an example of description of the active medium; we use the compact formalism found in Ref.17. Let N be the concentration of active centers, and let σ_{pa},σ_{pe} and σ_{sa},σ_{se} be effective absorption and emission cross-sections at the frequencies of the pump and the signal, respectively. In this approximation, the slope efficiency, under good operation conditions, may approach the quantum limit $\eta_o=\omega_s/\omega_p$. Therefore we should minimize the threshold.

In the constant intensity approximation, the threshold can be calculated using the compact formalism suggested by Ref. 17; then the optimal concentration N can be estimated. The rate equations can be written as follows:

$$\frac{dn_2}{dt}=(\sigma_{pa}j_p+\sigma_{sa}j_s)n_1-\left(\sigma_{pe}j_p+\sigma_{se}j_s+\frac{1}{\tau}\right)n_2, \quad \text{(A1)}$$

$$\frac{dn_1}{dt}=-(\sigma_{pa}j_p+\sigma_{sa}j_s)n_1+\left(\sigma_{pe}j_p+\sigma_{se}j_s+\frac{1}{\tau}\right)n_2, \quad \text{(A2)}$$

where n_1 and n_2 are relative populations at the lower and upper manifolds, and j is proportional to the intensities I_p, I_s of the pump and the signal:

$$j_p=I_s/(\hbar\omega_p), \quad j_s=I_p/(\hbar\omega_s). \quad \text{(A3)}$$

ASE is taken into account in Eqs. (A1) and (A2) with effective lifetime τ by Eq. (1).

The steady-state solution can be used,[6] for both the cw operation and pulses of the duration $\Delta t>\tau$:

$$n_1=\frac{1/\tau+\sigma_{pe}j_p+\sigma_{se}j_s}{1/\tau+(\sigma_{pa}+\sigma_{pe})j_p+(\sigma_{sa}+\sigma_{se})j_s}, \quad \text{(A4)}$$

$$n_2=\frac{\sigma_{pa}j_p+\sigma_{sa}j_s}{1/\tau+(\sigma_{pa}+\sigma_{pe})j_p+(\sigma_{sa}+\sigma_{se})j_s}. \quad \text{(A5)}$$

To express the estimate in compact form, we define the effective cross sections

$$\sigma_a=\frac{\sigma_{pa}\sigma_{se}-\sigma_{pe}\sigma_{sa}}{\sigma_{sa}+\sigma_{se}}, \quad \sigma_e=\frac{\sigma_{pa}\sigma_{se}-\sigma_{pe}\sigma_{sa}}{\sigma_{pa}+\sigma_{pe}}. \quad \text{(A6)}$$

We define the absorption at strong signal

$$A_0=N\frac{\sigma_{pa}\sigma_{se}-\sigma_{sa}\sigma_{pe}}{\sigma_{sa}+\sigma_{se}}=N\sigma_a \quad \text{(A7)}$$

and the gain at the strong pump

$$G_0=N\frac{\sigma_{pa}\sigma_{se}-\sigma_{sa}\sigma_{pe}}{\sigma_{pa}+\sigma_{pe}}=N\sigma_e. \quad \text{(A8)}$$

We define the effective saturation intensities

$$I_{po}=\frac{\hbar\omega_p/\tau}{\sigma_{pa}+\sigma_{pe}}, \quad I_{so}=\frac{\hbar\omega_p/\tau}{\sigma_{sa}+\sigma_{se}}. \quad \text{(A9)}$$

These intensities may depend on the geometry of the device through the lifetime τ by Eq. (1).

We define the cross-sectional invariants

$$U=\frac{(\sigma_{sa}+\sigma_{se})\sigma_{pa}}{\sigma_{pa}\sigma_{se}-\sigma_{sa}\sigma_{pe}}, \quad V=\frac{(\sigma_{pa}+\sigma_{pe})\sigma_{sa}}{\sigma_{pa}\sigma_{se}-\sigma_{sa}\sigma_{pe}}. \quad \text{(A10)}$$

Note that $U-V=1$.

Then, the absorption A of pump and the gain G of the signal can be expressed as follows:

$$A=(n_1\sigma_{pa}-n_2\sigma_{pe})N=A_0\frac{U+s}{1+p+s}, \quad \text{(A11)}$$

$$G=(n_2\sigma_{se}-n_1\sigma_{sa})N=G_0\frac{p-V}{1+p+s}, \quad \text{(A12)}$$

where

$$p=I_p/I_{po}, \quad s=I_s/I_{so} \quad \text{(A13)}$$

are the normalized intensities of pump and signal. Combining Eqs. (A11), (A12), and (A10), we obtain the identity

$$A/A_0 + G/G_0 = 1, \tag{A14}$$

which simplifies the deduction.

The round-trip gain $g=2Gh$ is determined by the output coupling θ and surface-scattering loss coefficient β. To get the normalized signal intensity s, the normalized intensity of pump should be

$$p = \frac{V + (1+s)G/G_0}{1 - G/G_0}. \tag{A15}$$

The threshold corresponds to $s=0$. This gives the estimate for the threshold pump intensity

$$I_{\mathrm{th}} = I_{\mathrm{po}} \frac{V + G/G_0}{1 - G/G_0}. \tag{A16}$$

We need this expression to estimate the threshold power.

APPENDIX B: CONCENTRATION AND THRESHOLD

Assume that in the multipass scheme, the pump comes through surface of area L^2, having loss μ. (We expect $\mu \ll 1$). At the threshold, the power $L^2\mu I_{\mathrm{th}}/2$ is lost at the reentries of the pump. (We assume that the pump makes a round trip inside the medium before leaving; so, only half of the threshold intensity corresponds to the incident or leaving pump.) The power $L^2 h A I_{\mathrm{th}}$ is absorbed within the medium. This assumption leads to the estimate for the threshold pump power

$$P_{\mathrm{th}} = (AL^2h + \mu L^2/2) I_{\mathrm{po}} \frac{V + G/G_0}{1 - G/G_0}. \tag{B1}$$

Using Eq. (A14), we rewrite estimate (B1) as

$$P_{\mathrm{th}} = (VG_0 + G)\left(\frac{A_0}{G_0}h + \frac{\mu/2}{G_0 - G}\right) L^2 I_{\mathrm{po}}. \tag{B2}$$

Consider the dependence of the threshold power on the concentration N. The ratio

$$b = \frac{A_0}{G_0} = \frac{\sigma_{\mathrm{pa}} + \sigma_{\mathrm{pe}}}{\sigma_{\mathrm{sa}} + \sigma_{\mathrm{se}}} \tag{B3}$$

does not depend on the concentration N, so we treat this ratio as constant. Also, the gain G is determined by parameters of the coupler, see Eq. (12). Then, the substitution of Eq. (A7) into Eq. (B2) gives:

$$P_{\mathrm{th}} = \left(N\sigma_{\mathrm{e}} + \frac{G}{V}\right)\left(bh + \frac{\mu/2}{N\sigma_{\mathrm{e}} - G}\right) L^2 V I_{\mathrm{po}}. \tag{B4}$$

The reasonable values of N are between $G/\sigma_{\mathrm{e}} = g/(2h\sigma_{\mathrm{e}})$ and $G/\sigma_{\mathrm{e}} = g/(2g\sigma_{\mathrm{e}}V)$. Note the values of the dimensionless parameter V by Eq. (A10). For example, at $I_{\mathrm{po}} = 204.5\ \mathrm{W/mm^2}$, $V=0.066$, $b=0.532$ (typical values for Yb:YAG, see Ref. 17), $L=3.4$ mm, $h=0.024$ mm, $\mu=0.01$, $g=0.03$, we have $G=g/(2h)=0.62/\mathrm{mm}$, $G/V=9.47/\mathrm{mm}$. The corresponding output power P_{s} versus product $N\sigma_{\mathrm{e}} = G_0$ is plotted in Fig. 3 with an intermediate curve.

For comparison, at the same graphic we plot also the cases with $\mu=0.02$, $g=0.05$ (thick curve) and $\mu=0.001$, $g=0.01$ (thin curve). Note that the threshold power depends on the round-trip gain. However, the threshold is not sensitive to what portion of power goes to the output signal (β) and what portion of power is just scattered (θ).

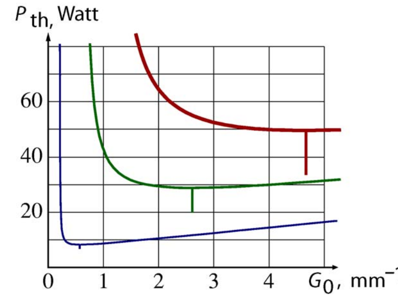

Fig. 3. (Color online) Threshold power estimated by Eq. (B4) as function of product $N\sigma_{\mathrm{e}}$, at $I_{\mathrm{po}}=204.5\ \mathrm{W/mm^2}$, $V=0.066$, $b=0.532$, $L=3.4$ mm, and $h=0.024$ mm for the cases $\mu=0.02$ and $g=0.05$ (thick curve), $\mu=0.01$ and $g=0.03$ (intermediate curve) and $\mu=0.001$ and $g=0.01$ (thin curve). Bars show the minimum by Eq. (B7) (upper tip) and the simple estimate (B9) (lower tip).

We define $X=N\sigma_{\mathrm{e}} - G$; then we can rewrite the output power as function of X:

$$P_{\mathrm{th}} = \left(bVhX + \frac{GU\mu/2}{X} + bUGh + V\mu/2\right) L^2 I_{\mathrm{po}}. \tag{B5}$$

(Identity $U=1+V$ was used.) The minimization of the threshold power with respect to the concentration N gives

$$X = \sqrt{\frac{GU}{bVh}}, \tag{B6}$$

which corresponds to the strong-pump gain

$$G_0 = \sigma_{\mathrm{e}} N = G + \sqrt{\frac{GU\mu}{2bVh}} = \left(1 + \sqrt{\frac{U\mu}{bVg}}\right) G, \tag{B7}$$

Substituting Eq. (B6) into Eq. (B5), we get

$$P_{\mathrm{th}} = (bUg/2 + \sqrt{gU\mu bV} + \mu V) L^2 I_{\mathrm{po}}. \tag{B8}$$

Note that in a typical case, b is of order of unity. We expect the pump launching loss μ to be small compared with the round-trip gain g; also, we expect $U \gg V$. Therefore we may neglect the two last terms in the parentheses in the right-hand side of Eq. (B8). Then, for the case $\sigma_{\mathrm{pa}} \gg \sigma_{\mathrm{pe}}$ and $\sigma_{\mathrm{se}} \gg \sigma_{\mathrm{sa}}$, we get the simple estimate of the threshold:

$$P_{\mathrm{th}} \approx \frac{gL^2 \hbar\,\omega/(2\tau)}{\sigma_{\mathrm{se}} - \sigma_{\mathrm{sa}}\sigma_{\mathrm{pe}}/\sigma_{\mathrm{pa}}} \approx \frac{\hbar\,\omega_{\mathrm{p}}}{\tau_0}\frac{L^2}{2\sigma_{\mathrm{se}}} g e^{GL}. \tag{B9}$$

The error of this simple estimate in shown in Fig. 2 with vertical bars. However, the estimate (B9) becomes more precise as we reduce the loss μ at the relaunching of the pump. The lower the loss at the surface is, the more transparent medium corresponds to the better performance. However, the threshold (and efficiency) of the laser is not very sensitive to the concentration N of the active centers. In the example with Yb in Fig. 2, the variation of the concentration for an amount of 50% around the optimal value determined by Eq. (B7) increases the threshold by only a few percent. For Nd active dopant, V is almost zero, and the maximal concentration of the active centers can be used without to drop the efficiency.

Note that if we remove the exponential factor from Eq. (B9), it becomes a special case of Eq. (1.36) from Ref. 22. (Case when the profile of pump perfectly matches the profile of the signal). The estimate (B9) can be obtained from Eq. (1.36) of Ref. 22 by scaling the relaxation time [Eq. (1)] due to the ASE.

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